1. Write an equation of the parabola that has vertex \((-1, 4)\), and that passes through \((1, -2)\).

\[ y = a(x + 1)^2 + 4 \]
\[ -2 = a(1 + 1)^2 + 4 \]
\[ a = -\frac{3}{2} \]
\[ y = -\frac{3}{2}(x + 1)^2 + 4 \]
or
\[ 3x^2 + 6x + 2y - 5 = 0 \]

2. Write an equation for the inverse of the function, and then graph both the function and its inverse.

\[ f(x) = x^2 - 6x \]

\[ x = y^2 - 6y \]
\[ x + 9 = y^2 - 6y + 9 \]
\[ x + 9 = (y - 3)^2 \]
\[ \pm \sqrt{x + 9} = y - 3 \]
\[ y = 3 \pm \sqrt{x + 9} \]

3. Write an equation for the tangent to the graph of \( y = x^3 - 5x - 1 \) at \( x = 2 \).

\[ y' = 3x^2 - 5 \]
At \( x = 2 \), \( y = (2)^3 - 5(2) - 1 = -3 \), \( m = 3(2)^2 - 5 = 7 \)
\[ y = mx + b \]
\[ -3 = 7(2) + b \]
\[ b = -17 \]
\[ y = 7x - 17 \]

4. Identify the critical points (zeros, \( y \)-intercept, local maxima, local minima, and points of inflection) and label them on a graph of the function.

\[ f(x) = 2x^3 - 3x^2 - 36x + 145 \]
\[ f'(x) = 6x^2 - 6x - 36 = 6(x - 3)(x + 2) \]
\[ f''(x) = 12x - 6 = 6(2x - 1) \]
zeros \((-5, 0)\)
\( y \)-intercept \(= (0, 145) \)
local minimum \(= (3, 64) \)
local maximum \(= (-2, 189) \)
point of inflection \(= \left(\frac{1}{2}, 126\frac{1}{2}\right) \)
5. Graph the function and describe the domain, range, asymptotes, and intercepts:

\[ f(x) = \frac{x^3 - x^2 - 6x}{x^2 - 4} \]

\[ \frac{x}{x^2 - 4} \left( x^3 - x^2 - 6x \right) = x - 1 + \frac{-2x - 4}{x^2 - 4} \]

The domain is \( x \neq 2, -2 \)

The range is \( y \in \mathbb{R} \)

x-intercepts = 0, 3

y-intercept = 0

asymptotes: \( x = 2, y = x - 1 \)

6. Find the centre and radius of the circle.

\[ x^2 + y^2 + 8x - 10y = 0 \]

\[ x^2 + 8x + 16 + y^2 - 10y + 25 = 16 + 25 \]

\( (x + 4)^2 + (y - 5)^2 = 41 \)

centre \(-4, 5\), radius \( \sqrt{41} \)

7. Write the equation for a circle that has A(4,7) and B(-2,1) as two endpoints of a diameter.

7*. Write an equation for the circle that passes through the points A(-3, 6), B(1, 8), and C(5, 0).

Chord BC: midpoint (3, 4), slope \( \frac{8}{4} = -2 \), perpendicular \( \frac{1}{2} \)

Perpendicular bisector of AB:

\[ y = \frac{5}{2}x + b \]

\[ 4 = \frac{5}{2}(3) + b \]

\[ b = \frac{5}{2} \]

\[ y = \frac{5}{2}x + \frac{5}{2} \]

Chord AB: midpoint (-1, 7), slope \( \frac{2}{4} = \frac{1}{2} \), perpendicular -2

\[ 7 = -2(1) + b \]

\[ b = 5 \]

\[ y = -2x + 5 \]

Intersection (subtract two equations, and substitute result) \( \ldots \)

\[ 0 = -\frac{5}{2}x + \frac{5}{2} \]

\[ x = 1 \]

\[ y = -2(1) + 5 = 3 \]

centre \((1, 3)\), radius 5 (distance from centre to point B)

\[ (x - 1)^2 + (y - 3)^2 = 25 \]

equation \( \ldots \)

\[ x^2 + y^2 - 2x - 6y + 9 = 25 \]

\[ x^2 + y^2 - 2x - 6y - 15 = 0 \]
8. Solve $\triangle ABC$ with $AB = 8$ cm, $BC = 5$ cm, and $\angle A = 30^\circ$.

9. Prove the identity:

Left Side:

$$\sin 2x = \frac{2 \tan x}{1 + \tan^2 x}$$

$$= 2 \sin x \cos x$$

Right Side:

$$= \frac{2 \sin x}{1 + \frac{\sin^2 x}{\cos^2 x}}$$

$$= \frac{2 \sin x \cos^2 x + \sin^2 x}{\cos x}$$

$$= \frac{2 \sin x \cos^2 x}{\cos x}$$

$$= 2 \sin x \cos x$$

10. Graph at least two cycles of

$$f(\theta) = -3 \sin(2(\theta + \pi)) + 1$$
11. Write a function that describes the given graph:

\[
\begin{align*}
&\text{period } = 2\pi \\
&\text{amplitude } = 2 \\
&\text{horizontal disp. } = \frac{2\pi}{3} \\
&\text{vertical disp. } = -2 \\
&\text{Note: } (2\pi)(b) = 2\pi \\
&b = 1 \\
&f(\theta) = 2\cos(\theta - \frac{2\pi}{3}) - 2
\end{align*}
\]

12. A band wants to make a CD with 6 songs, drawing on their repertoire of 9 songs. How many ways could they make the CD? (The order of the songs matters.)

If order matters to the band \(9 \times 8 \times 7 \times 6 \times 5 \times 4 = 60480\)

13. What is the probability of rolling a sum of 8 when rolling two dice?

2, 6 or 3, 5 or 4, 4 or 5, 3 or 6, 2 ... 5 ways out of 36 = 5/36

14. What is the probability of 3 boys winning the 3 raffle prizes in a class of 14 girls and 12 boys?

\[
\frac{12}{26} \times \frac{11}{25} \times \frac{10}{24} = \frac{1320}{15600} = \frac{11}{130}
\]

15. How many possible committees of 3 could be chosen to represent a class of 26?

\[
\binom{26}{3} = 2600
\]

16. What is the probability of rolling four fives if you roll a die six times?

\[
P(5) = \frac{1}{6}, \quad P'(5) = \frac{5}{6}
\]

\[
\frac{6\binom{3}{5} (\frac{1}{6})^4 (\frac{5}{6})^2}{\binom{6}{4}} = \frac{15 \times \frac{1}{1296} \times \frac{25}{36}}{15600} = \frac{125}{15552}
\]

17. What is the number of ways the letters in the word SUCCESSFUL can be arranged?

\[
\frac{10!}{3!2!2!} = 151200
\]

17* How does the number of arrangements change if the first letter must be a vowel?

\[
\frac{3 \times 9!}{3!2!2!} = 45360
\]

There are 3/10 the arrangements because 3/10 of the letters are vowels.
18. What is the fourth term in the expansion of \((a + b)^1\)?

\[ \binom{1}{3} a^8 b^3 = 165 a^8 b^3 \]

* What is the fourth term in the expansion of \((3x - 2)^11\)?

\[ \binom{11}{3} (3x)^8 (-2)^3 = -8660520 x^8 \]

19. What is the seventh number in the row of Pascal’s triangle beginning 1 13 78 …?

\[ \binom{13}{6} = 1716 \]

20. For the sequence, give both a recursive rule and an explicit rule, and find the tenth term.

\{3, -6, 12, -24, \ldots \}

Recursive … \[ t_1 = 3, t_{n+1} = -2t_n, n \in \mathbb{N} \]

Explicit … \[ a = 3, r = -2, \ t_n = ar^{n-1} = 3(-2)^{n-1}, n \in \mathbb{N} \]

\[ t_{10} = 3(-2)^{10-1} = -1536 \]

21. Expand to show at least the first two terms and calculate the sum of the full series:

\[ \sum_{j=2}^{45} 2(j - 24) = 2(2 - 24) + 2(3 - 24) + \ldots + 2(45 - 24) = -44 \]

\[ \frac{n}{2} (a + t_n) = \frac{44}{2} (-44 + 42) = -44 \]

22. Represent the series using sigma notation and calculate the sum:

\[ a = 207, \ d = -3 \]

\[ t_n = a + (n - 1)d \]

\[ \sum_{n=1}^{121} (210 - 3n) \]

\[ 207 + 204 + 201 + 198 + \ldots + (-153) = \frac{121}{2} \times [207 + (-153)] = 3267 \]

23. Represent the series using sigma notation and calculate the sum:

\[ a = 54, \ r = \frac{16}{24} = \frac{2}{3} \]

\[ t_n = ar^{n-1} = 54 \left(\frac{2}{3}\right)^{n-1} = 54 \left(\frac{2}{3}\right) \]

\[ \sum_{n=1}^{\infty} 8 \left(\frac{2}{3}\right)^n = \frac{a}{1 - r} = \frac{54}{1 - \frac{2}{3}} = 162 \]

24. An investment increased 3.2% in the first year, 4.5% in the second, 1.3% in the third, and 14.7% in the fourth. What is the average annual increase?

Let \( P = \text{principal} \)

Final value = \( P \times 1.032 \times 1.045 \times 1.013 \times 1.147 = 1.253P \)

\( 1.253^{\frac{4}{4}} = 1.058 \)

The average increase is \( 1.058 - 1 = 5.8\% \)
25. Use first principles to find the derivative of \( f(x) = 5x^3 \).

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
= \lim_{h \to 0} \frac{5(x+h)^3 - 5x^3}{h} \\
= 5 \times \left[ \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} \right] \\
= 5 \times \left[ \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \right] \\
= 5 \times \left[ \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3}{h} \right] \\
= 5 \times \left[ \lim_{h \to 0} 3x^2 + 3xh + h^2 \right] \\
= 5(3x^2) \\
= 15x^2
\]

26. If \( x = \log 3 \) and \( y = \log 5 \), write the following expression as a single logarithm: \( 2x + 1 - y \)

\[
2 \log 3 + \log 10 - \log 5 \\
= \log \frac{30}{5} \\
= \log 6
\]

* Given \( \log 3 = p \), \( \log 5 = q \), and \( \log 7 = r \), express \( \log 42 \) in terms of \( p \), \( q \), and \( r \).

\[
\log 42 = \log \left( \frac{3 \times 10 \times 7}{5} \right) = \log 3 + \log 10 + \log 7 - \log 5 = p + 1 + r - q
\]

27. Determine a function with \( y \)-intercept \((0,90)\), and \( x \)-intercepts \((-3,0)\), \((2,0)\), and \((3,0)\).

\[
y = k(x - 3)(x - 2)(x + 3) \\
90 = k(-3)(-2)(3) \\
k = 18 \\
f(x) = 5(x - 3)(x - 2)(x + 3)
\]
28. Evaluate: 
\[(1 - i\sqrt{3})^{11}\]
\[(1 - i\sqrt{3})^2 = 1 - 2i\sqrt{3} - 3 = -2 - 2i\sqrt{3}\]
\[(1 - i\sqrt{3})^4 = (\sqrt{-2 - 2i\sqrt{3}})^2 = 4 + 8i\sqrt{3} - 12 = -8 + 8i\sqrt{3}\]
\[(1 - i\sqrt{3})^8 = (\sqrt[4]{-8 + 8i\sqrt{3}})^2 = 64 - 128i\sqrt{3} - 192 = -128 - 128i\sqrt{3}\]
\[(1 - i\sqrt{3})^3 = (\sqrt{-2 - 2i\sqrt{3}})(1 - i\sqrt{3}) = -2 + 2i\sqrt{3} - 2i\sqrt{3} - 6 = -8\]
\[(1 - i\sqrt{3})^{11} = -8(128 - 128i\sqrt{3}) = 1024 + 1024i\sqrt{3}\]
or
\[(1 - i\sqrt{3})^{11} = \left[|1 - i\sqrt{3}| \text{cis} \frac{5\pi}{3}\right]^{11}\]
\[= 2^{11} \times \text{cis} \frac{55\pi}{3}\]
\[= 2048 \times \text{cis} \left(\frac{\pi}{3}\right)\]
\[= 2048\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\]
\[= 1024 + 1024i\sqrt{3}\]
or
\[(1 - i\sqrt{3})^{11} = \sum_{n=0}^{11} \binom{11}{n} (-1)^n (i\sqrt{3})^n\]
\[= 11C_0(1)^{11}(-i\sqrt{3})^0 + 11C_1(1)^{10}(-i\sqrt{3})^1 + 11C_2(1)^9(-i\sqrt{3})^2 + \ldots + 11C_{11}(1)^0(-i\sqrt{3})^{11}\]
\[= 1 - 11i\sqrt{3} + 55(-3) - 165(-3)(-i\sqrt{3}) + 330(9) - 462(9)(-i\sqrt{3}) + 462(-27)\]
\[= 330(-27)(-i\sqrt{3}) + 165(81) - 55(81)(-i\sqrt{3}) + 11(-243) - 1(-243)(-i\sqrt{3})\]
\[= 1(165) + 2970 - 12474 + 13365 - 2673\]
\[+i\sqrt{3}(-11 + 495 - 4158 + 8910 - 4455 + 243)\]
\[= 1024 + 1024i\sqrt{3}\]

29. Find all real and non-real roots:
\[x^3 - 17x + 40 = 0\]
\[(x + 5)(x^2 - 5x + 8) = 0\]
\[x^3 - 17x + 40 = 0\]
\[x = -5, x = \frac{5 \pm \sqrt{-7}}{2} = \frac{5 \pm i\sqrt{7}}{2}\]

30. For what real values of \(k\) is there no solution to this equation?
\[x^2 - 7x - k = 0\]
\[(-7)^2 - 4(1)(-k) < 0\]
\[49 + 4k < 0\]
\[k < -\frac{49}{4}\]

31. Solve:
\[\frac{3}{2}x + y = 8\]
\[-2.1x + 0.5y = -7.4\]
32. Solve $\sin x + \sin 2x = 0$
   for $x \in [-2\pi, 2\pi]$ $\sin x + 2\sin x \cos x = 0$
   $\sin x(1 + 2\cos x) = 0$
   $\sin x = 0$ $1 + 2\cos x = 0$
   $x = 0, \pi, -\pi, 2\pi, -2\pi$ $\cos x = -\frac{1}{2}$
   $x = \frac{2\pi}{3}, -\frac{2\pi}{3}, \frac{4\pi}{3}, -\frac{4\pi}{3}$
   $x = 0, \pi, -\pi, 2\pi, -2\pi, \frac{2\pi}{3}, -\frac{2\pi}{3}, \frac{4\pi}{3}, -\frac{4\pi}{3}$

33. Solve:
   \[
   \frac{7}{x+1} - \frac{x}{x+4} = 2
   \]
   $7x + 28 - x^2 = x^2 + 10x + 8$
   $0 = 3x^2 + 4x - 20$
   $0 = (3x + 10)(x - 2)$
   $x = -\frac{10}{3}, 2$

34. Solve
   \[4x^3 + 13x + 12 \geq 20x^2\]
   \[4x^3 - 20x^2 + 13x + 12 \geq 0\]
   \[(x - 4)(4x^2 - 4x - 3) \geq 0\]
   \[(x - 4)(2x - 3)(2x + 1) \geq 0\]
   $x \in [-\frac{1}{2}, \frac{3}{2}] \cup [4, \infty)$

35. Solve $1 \leq -2x + 5 < 7$
   $x \in (-1, 2] \cup [3, 6)$

or

Critical points at …
1 = $-2x + 5$ … $x = 2$
1 = $-(-2x + 5)$ … $x = 3$
$-2x + 5 = 7$ … $x = -1$
$-(-2x + 5) = 7$ … $x = 6$
Test original inequality at each critical point and in each interval …
x = -2 is false, $x = -1$ is false, $x = 0$ is true, $x = 2$ is true, $x = 2.5$ is false, $x = 3$ is true, $x = 5$ is true, $x = 6$ is false, $x = 7$ is false

$-1 < x \leq 2$ or $3 \leq x < 6$
36. Solve
\[ \frac{x^2 + 3x + 3}{x + 3} \leq 1 \]

\[ \frac{x^2}{x + 3} - \frac{x + 3}{x + 3} \leq 0 \]
\[ \frac{x^2 + 2x}{x + 3} \leq 0 \]
\[ \frac{x(x + 2)}{x + 3} \leq 0 \]
\[ x \in (-\infty, -3) \cup [-2, 0] \]

or

\[ \frac{x^2 + 3x + 3}{x + 3} - \frac{x + 3}{x + 3} \leq 0 \]
\[ \frac{x^2 + 2x}{x + 3} \leq 0 \]
\[ \frac{x(x + 2)}{x + 3} \leq 0 \]

Critical points where numerator or denominator are zero …
\[ x = 0, x = -2, x = -3 \]

Test original inequality at each critical point and in each interval …
\[ x = -4 \text{ is true, } x = -3 \text{ is false, } x = -2.5 \text{ is false, } x = -2 \text{ is true, } x = -1 \text{ is true, } x = 0 \text{ is true, } x = 1 \text{ is false} \]

\[ x < -3 \text{ or } -2 \leq x \leq 0 \]
37. Solve $2x = \sqrt{6 - 5x}$

$(2x)^2 = 6 - 5x$

$4x^2 + 5x - 6 = 0$ checking shows $x \neq -2$, so $x = \frac{3}{4}$

$x = \frac{3}{4}, -2$

38. Solve:

$9^{5-2x} = \frac{1}{27^{x-3}}$

$(3^2)^{5-2x} = (3^{-3})^{x-3}$

$3^{10-4x} = 3^{9-3x}$

$10 - 4x = 9 - 3x$

$x = 1$

39. Solve:

$log_5 x^2 = log_5 25 + log_5 3x$

$log_5 x^2 = log_5 75x$

$2 log_5 x = 2 + log_5 3x$

$log_5 x^2 = log_5 75x$

$x^2 = 75x$

$x = 0, 75$

$x \neq 0$

$x = 75$

40. Solve $e^{2x} - 10 = 3e^x$

Let $y = e^x$

$e^{2x} - 3e^x - 10 = 0$

$y^2 - 3y - 10 = 0$

$(y - 5)(y + 2) = 0$

$y = 5, -2$

$e^x = 5, -2$

but $e^x > 0$

$e^x = 5$

$x = \ln 5$