1. Write an equation of the parabola that has vertex (-1, 4) , and that passes through (1, -2).

$y = a(x+1)^2 + 4$

$-2 = a(1+1)^2 + 4$

$a = -\frac{3}{2}$

$y = -\frac{3}{2}(x+1)^2 + 4$

or $3x^2 + 6x + 2y - 5 = 0$

2. Write an equation for the inverse of the function, and then graph both the function and its inverse.

$f(x) = x^2 - 6x$

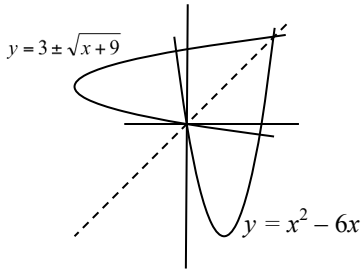
$x = y^2 - 6y$

$x + 9 = y^2 - 6y + 9$

$x + 9 = (y-3)^2$

$\pm\sqrt{x+9} = y - 3$

$y = 3 \pm \sqrt{x+9}$



3. Write an equation for the tangent to the graph of $y = x^3 - 5x - 1$ at $x = 2$.

$y' = 3x^2 - 5$

At $x = 2$, $y = (2)^3 - 5(2) - 1 = -3$, $m = 3(2)^2 - 5 = 7$

$y = mx + b$

$-3 = 7(2) + b$

$b = -17$

$y = 7x - 17$

4. Identify the critical points (zeros, y-intercept, local maxima, local minima, and points of inflection) and label them on a graph of the function.

$f(x) = 2x^3 - 3x^2 - 36x + 145$

$f(x) = 2x^3 - 3x^2 - 36x + 145 = (x+5)\left(2x^2 - 13x + 29\right)$

$f'(x) = 6x^2 - 6x - 36 = 6(x-3)(x+2)$

$f''(x) = 12x - 6 = 6(2x-1)$

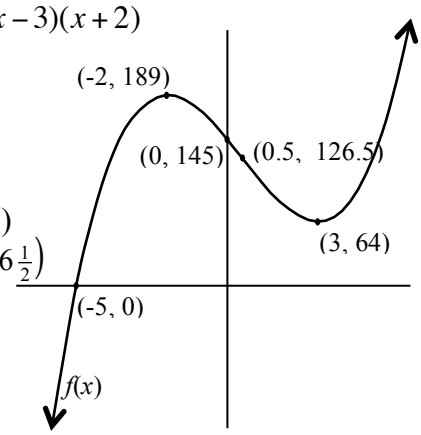
zeros …(-5,0)

y-intercept = (0, 145)

local minimum = (3, 64)

local maximum = (-2, 189)

point of inflection = $\left(\frac{1}{2}, 126\frac{1}{2}\right)$



5. Graph the function and describe the domain, range, asymptotes, and intercepts:

$$f(x) = \frac{x^3 - x^2 - 6x}{x^2 - 4}$$

$$\begin{array}{r|rrrr} & x & -1 & & \\ \hline x^2-4 & x^3 & -x^2 & -6x & +0 \\ & x^3 & & -4x & \\ \hline & & -x^2 & -2x & \\ & & -x^2 & & +4 \\ \hline & & & -2x & -4 \end{array}$$

$$f(x) = \frac{x^3 - x^2 - 6x}{x^2 - 4} = \frac{x(x-3)(x+2)}{(x-2)(x+2)} = x - 1 + \frac{-2x-4}{x^2-4}$$

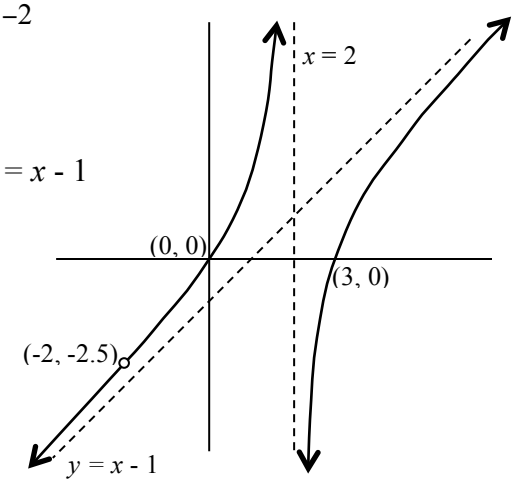
The domain is $x \neq 2, -2$

The range is $y \in \mathbf{R}$

x-intercepts = 0, 3

y-intercept = 0

asymptotes: $x = 2$, $y = x - 1$



6. Find the centre and radius of the circle.

$$x^2 + y^2 + 8x - 10y = 0$$

$$x^2 + y^2 + 8x - 10y = 0$$

$$x^2 + 8x + 16 + y^2 - 10y + 25 = 16 + 25$$

$$(x+4)^2 + (y-5)^2 = 41$$

centre (-4, 5), radius $\sqrt{41}$

7. Write the equation for a circle that has A(4,7) and B(-2,1) as two endpoints of a diameter.

Centre = midpoint of AB = M(1,4)

Radius = length of MA = $\sqrt{(1-4)^2 + (4-7)^2} = \sqrt{18}$

Equation … $(x-1)^2 + (y-4)^2 = 18$

one could expand if desired … $x^2 - 2x + y^2 - 8y - 1 = 0$

7*. Write an equation for the circle that passes through the points A(-3, 6), B(1, 8), and C(5, 0).

Chord BC: midpoint (3, 4), slope $\frac{8}{-4} = -2$, perpendicular $\frac{1}{2}$

Perpendicular bisector of AB:

$y = mx + b$

$4 = \frac{1}{2}(3) + b$

$b = \frac{5}{2}$

$y = \frac{1}{2}x + \frac{5}{2}$

Chord AB: midpoint (-1, 7), slope $\frac{2}{4} = \frac{1}{2}$, perpendicular -2

$7 = -2(1) + b$

$b = 5$

$y = -2x + 5$

Intersection (subtract two equations, and substitute result) …

$0 = -\frac{5}{2}x + \frac{5}{2}$

$x = 1$

$y = -2(1) + 5 = 3$

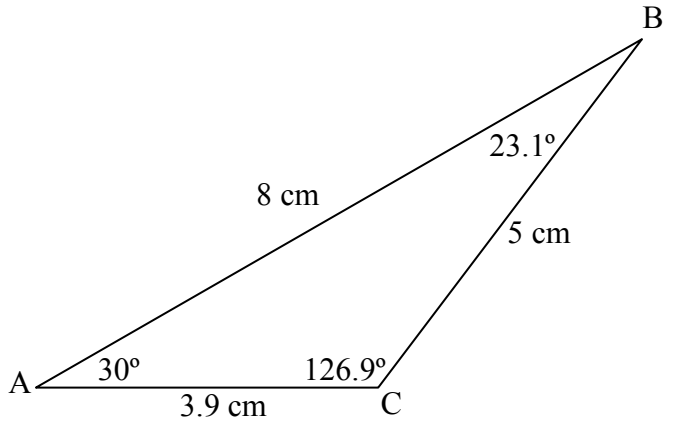
centre (1, 3), radius 5 (distance from centre to point B)

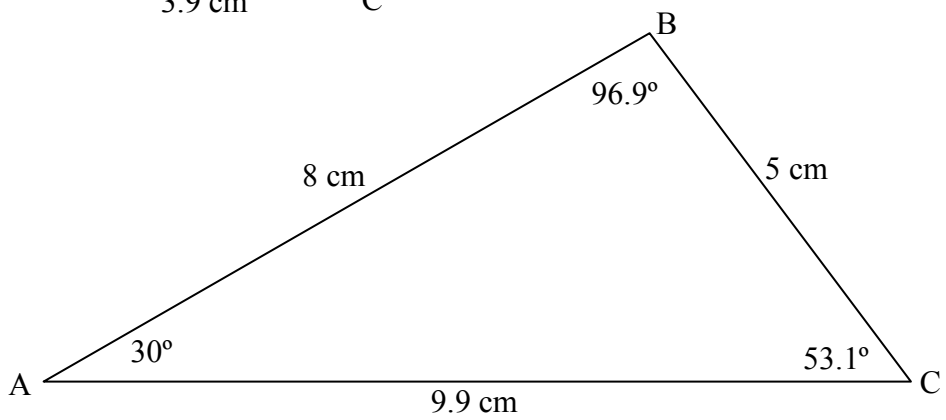
$$(x-1)^2 + (y-3)^2 = 25$$

equation … $x^2 - 2x + 1 + y^2 - 6y + 9 = 25$

$$x^2 + y^2 - 2x - 6y - 15 = 0$$

8. Solve ΔABC with AB = 8 cm, BC = 5 cm, and ∠ A = 30º.





9. Prove the identity:

$$\sin 2x = \frac{2\tan x}{1+\tan^2 x}$$

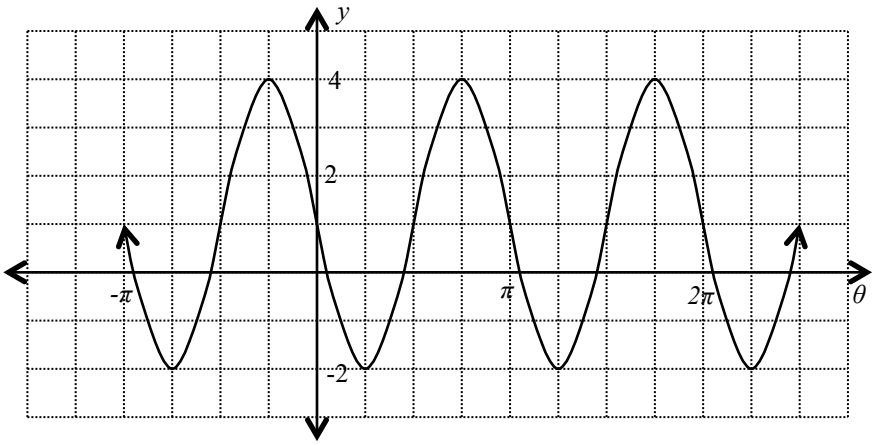
Left Side:

$$\sin 2x$$
$$= 2\sin x\cos x$$

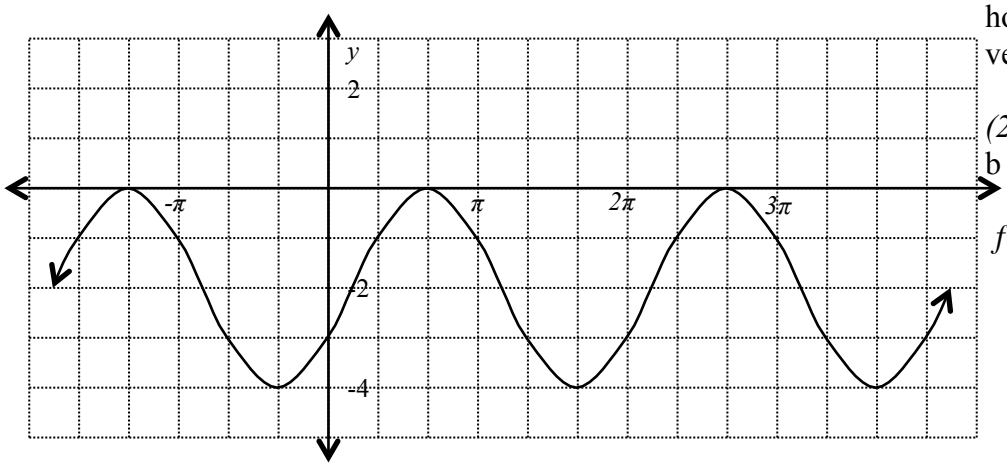
Right Side:

$$\frac{2\tan x}{1+\tan^2 x}$$
$$= \frac{\frac{2\sin x}{\cos x}}{1+\frac{\sin^2 x}{\cos^2 x}}$$
$$= \frac{2\sin x}{\cos x} \div \frac{\cos^2 x+\sin^2 x}{\cos^2 x}$$
$$= \frac{2\sin x}{\cos x} \times \frac{\cos^2 x}{1}$$
$$= 2\sin x\cos x$$

10. Graph at least two cycles of $f(\theta) = -3\sin 2(\theta+\pi)+1$



11. Write a function that describes the given graph:



period = 2π
amplitude = 2
horizontal disp. = $2\pi/3$
vertical disp. = -2

$(2\pi)(b) = 2\pi$
b = 1

$f(\theta) = 2\cos(\theta - \frac{2\pi}{3}) - 2$

12. A band wants to make a CD with 6 songs, drawing on their repertoire of 9 songs. How many ways could they make the CD? (The order of the songs matters.)

If order matters to the band …9 x 8 x 7 x 6 x 5 x 4 = 60480

13. What is the probability of rolling a sum of 8 when rolling two dice?

2,6 or 3,5 or 4,4 or 5,3 or 6,2 …5 ways out of 36 = 5/36

14. What is the probability of 3 boys winning the 3 raffle prizes in a class of 14 girls and 12 boys?

$$\frac{12}{26} \times \frac{11}{25} \times \frac{10}{24} = \frac{1320}{15600} = \frac{11}{130}$$

15. How many possible committees of 3 could be chosen to represent a class of 26?

$${}_{26}C_3 = 2600$$

16. What is the probability of rolling four fives if you roll a die six times?

$P(5) = 1/6$, $P'(5) = 5/6$

$${}_6C_4\left(\tfrac{1}{6}\right)^4\left(\tfrac{5}{6}\right)^2 = 15 \times \tfrac{1}{1296} \times \tfrac{25}{36} = \tfrac{125}{15552}$$

17. What is the number of ways the letters in the word SUCCESSFUL can be arranged?

$$\frac{10!}{3!2!2!} = 151200$$

17* How does the number of arrangements change if the first letter must be a vowel?

$$\frac{3 \times 9!}{3!2!2!} = 45360$$

There are 3/10 the arrangements because 3/10 of the letters are vowels.

18. What is the fourth term in the expansion of $(a+b)^{11}$?
* What is the fourth term in the expansion of $(3x-2)^{11}$?

$${}_{11}C_3 a^8 b^3$$
$$= 165a^8b^3$$

* … $${}_{11}C_3(3x)^8(-2)^3$$
$$= 165(6561x^8)(-8)$$
$$= -8660520x^8$$

19. What is the seventh number in the row of Pascal's triangle beginning 1 13 78 …?

$${}_{13}C_0 = 1, {}_{13}C_1 = 13, {}_{13}C_2 = 78, \ldots {}_{13}C_6 = 1716$$

20. For the sequence, give both a recursive rule and an explicit rule, and find the tenth term.
{3, -6, 12, -24, …}

recursive … $t_1 = 3, t_{n+1} = -2t_n, n \in \mathbf{N}$
explicit … $a = 3$, $r = -2$, $t_n = ar^{n-1} = 3(-2)^{n-1}, n \in \mathbf{N}$
$t_{10} = 3(-2)^{10-1} = -1536$

21. Expand to show at least the first two terms and calculate the sum of the full series:

$$\sum_{j=2}^{45} 2(j-24)$$

$$\sum_{j=2}^{45} 2(j-24) = 2(2-24) + 2(3-24) + \ldots + 2(45-24)$$
$$= -44 - 42 - \ldots + 42$$
$$= \tfrac{n}{2}(a + t_n) = \tfrac{44}{2}(-44 + 42) = -44$$

22. Represent the series using sigma notation and calculate the sum:

207 + 204 + 201 + 198 + … + (-153)

$a = 207$, $d = -3$

$$t_n = a + (n-1)d$$
$$t_n = 207 - 3(n-1) = 210 - 3n$$
$$-153 = 210 - 3n$$
$$n = 121$$

$$\sum_{n=1}^{121} (210 - 3n)$$
$$= \tfrac{121}{2} \times [207 + (-153)]$$
$$= 3267$$

23. Represent the series using sigma notation and calculate the sum:

54 + 36 + 24 + 16 + …

$a = 54$, $r = \frac{16}{24} = \frac{2}{3}$

$$t_n = ar^{n-1} = 54\left(\tfrac{2}{3}\right)^{n-1} = 54\left(\tfrac{3}{2}\right)\left(\tfrac{2}{3}\right)^n = 81\left(\tfrac{2}{3}\right)^n$$

$$\sum_{n=1}^{\infty} 81\left(\tfrac{2}{3}\right)^n = \frac{a}{1-r} = \frac{54}{1-\frac{2}{3}} = 162$$

24. An investment increased 3.2% in the first year, 4.5% in the second, 1.3% in the third, and 14.7% in the fourth. What is the average annual increase?

Let P = prinicipal
Final value = P x 1.032 x 1.045 x 1.013 x 1.147 = 1.253P
$1.253^{1/4} = 1.058$
The average increase is 1.058 – 1 = 5.8%

25. Use first principles to find the derivative of $f(x) = 5x^3$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{5(x+h)^3 - 5x^3}{h}$$
$$= 5 \times \left[\lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} \right]$$
$$= 5 \times \left[\lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \right]$$
$$= 5 \times \left[\lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3}{h} \right]$$
$$= 5 \times \left[\lim_{h \to 0} 3x^2 + 3xh + h^2 \right]$$
$$= 5(3x^2)$$
$$= 15x^2$$

26. If $x = \log 3$ and $y = \log 5$, write the following expression as a single logarithm: $2x + 1 - y$

$2 \log 3 + \log 10 - \log 5$
$= \log \frac{90}{5}$
$= \log 18$

* Given $\log 3 = p$, $\log 5 = q$, and $\log 7 = r$, express $\log 42$ in terms of p, q, and r.

$\log 42 = \log \left(\frac{3 \times 10 \times 7}{5} \right) = \log 3 + \log 10 + \log 7 - \log 5 = p + 1 + r - q$

27. Determine a function with y-intercept (0,90), and x-intercepts (-3,0), (2,0), and (3,0).

$y = k(x-3)(x-2)(x+3)$
$90 = k(-3)(-2)(3)$
$90 = 18k$
$k = 5$
$f(x) = 5(x-3)(x-2)(x+3)$

28. Evaluate: $\left(1-i\sqrt{3}\right)^{11}$

$$\left(1-i\sqrt{3}\right)^2 = 1-2i\sqrt{3}-3 = -2-2i\sqrt{3}$$

$$\left(1-i\sqrt{3}\right)^4 = \left(-2-2i\sqrt{3}\right)^2 = 4+8i\sqrt{3}-12 = -8+8i\sqrt{3}$$

$$\left(1-i\sqrt{3}\right)^8 = \left(-8+8i\sqrt{3}\right)^2 = 64-128i\sqrt{3}-192 = -128-128i\sqrt{3}$$

$$\left(1-i\sqrt{3}\right)^3 = \left(-2-2i\sqrt{3}\right)\left(1-i\sqrt{3}\right) = -2+2i\sqrt{3}-2i\sqrt{3}-6 = -8$$

$$\left(1-i\sqrt{3}\right)^{11} = -8\left(-128-128i\sqrt{3}\right) = 1024+1024i\sqrt{3}$$

or

$$\left(1-i\sqrt{3}\right)^{11}$$

$$= \left[\left|1-i\sqrt{3}\right| cis\,\tfrac{5\pi}{3}\right]^{11}$$

$$= 2^{11} \times cis\,\tfrac{55\pi}{3}$$

$$= 2048 cis\,\tfrac{\pi}{3}$$

$$= 2048\left(\tfrac{1}{2}+\tfrac{\sqrt{3}}{2}i\right)$$

$$= 1024+1024i\sqrt{3}$$

or

$$\left(1-i\sqrt{3}\right)^{11}$$

$$= {}_{11}C_0(1)^{11}\left(-i\sqrt{3}\right)^0 + {}_{11}C_1(1)^{10}\left(-i\sqrt{3}\right)^1 + {}_{11}C_2(1)^9\left(-i\sqrt{3}\right)^2 + \ldots + {}_{11}C_{11}(1)^0\left(-i\sqrt{3}\right)^{11}$$

$$= 1-11i\sqrt{3}+55(-3)-165(-3)\left(-i\sqrt{3}\right)+330(9)-462(9)\left(-i\sqrt{3}\right)+462(-27)$$

$$-330(-27)\left(-i\sqrt{3}\right)+165(81)-55(81)\left(-i\sqrt{3}\right)+11(-243)-1(-243)\left(-i\sqrt{3}\right)$$

$$= (1-165+2970-12474+13365-2673$$

$$+i\sqrt{3}(-11+495-4158+8910-4455+243)$$

$$= 1024+1024i\sqrt{3}$$

29. Find all real and non-real roots:

$$x^3-17x+40=0$$

$$x^3-17x+40=0$$

$$(x+5)\left(x^2-5x+8\right)=0$$

$$x=-5, x=\frac{5\pm\sqrt{-7}}{2} = \tfrac{5}{2}\pm\tfrac{i\sqrt{7}}{2}$$

30. For what real values of k is there no solution to this equation?

$$x^2-5x=2x+k$$

$$x^2-7x-k=0$$

$$(-7)^2-4(1)(-k)<0$$

$$49+4k<0$$

$$k<-\tfrac{49}{4}$$

31. Solve:

$$\tfrac{3}{2}x+y=8$$
$$-2.1x+0.5y=-7.4$$

(4, 2)

32. Solve $\sin x + \sin 2x = 0$ for $x \in [-2\pi, 2\pi]$

$\sin x + 2\sin x \cos x = 0$

$\sin x(1 + 2\cos x) = 0$

$\sin x = 0$

$x = 0, \pi, -\pi, 2\pi, -2\pi$

$1 + 2\cos x = 0$

$\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}, -\frac{2\pi}{3}, \frac{4\pi}{3}, -\frac{4\pi}{3}$

$x = 0, \pi, -\pi, 2\pi, -2\pi, \frac{2\pi}{3}, -\frac{2\pi}{3}, \frac{4\pi}{3}, -\frac{4\pi}{3}$

33. Solve:

$$\frac{7}{x+1} - \frac{x}{x+4} = 2$$

$$\frac{7}{x+1} - \frac{x}{x+4} = 2$$

$7x + 28 - x^2 - x = 2x^2 + 10x + 8$

$0 = 3x^2 + 4x - 20$

$0 = (3x + 10)(x - 2)$

$x = -\frac{10}{3}, 2$

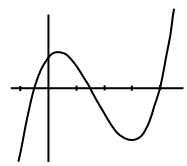
34. Solve $4x^3 + 13x + 12 \geq 20x^2$

$4x^3 - 20x^2 + 13x + 12 \geq 0$

$(x - 4)\left(4x^2 - 4x - 3)\right) \geq 0$

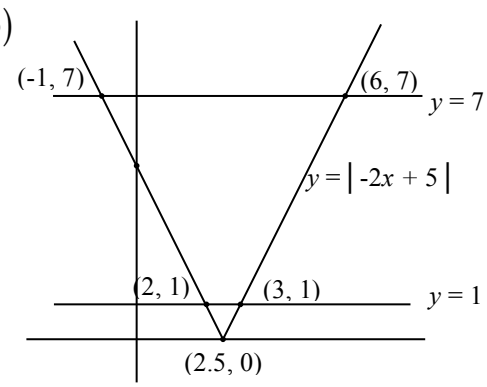
$(x - 4)(2x - 3)(2x + 1) \geq 0$

$x \in \left[-\frac{1}{2}, \frac{3}{2}\right] \cup [4, \infty)$



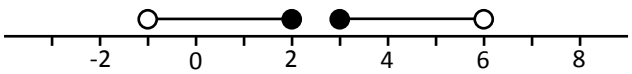
35. Solve $1 \leq |-2x + 5| < 7$

$x \in (-1, 2] \cup [3, 6)$



or

Critical points at …
$1 = -2x + 5$ … $x = 2$
$1 = -(-2x + 5)$ … $x = 3$
$-2x + 5 = 7$ … $x = -1$
$-(-2x + 5) = 7$ … $x = 6$
Test original inequality at each critical point and in each interval …
$x = -2$ is false, $x = -1$ is false, $x = 0$ is true, $x = 2$ is true, $x = 2.5$ is false, $x = 3$ is true, $x = 5$ is true, $x = 6$ is false, $x = 7$ is false



$-1 < x \leq 2$ or $3 \leq x < 6$

36. Solve

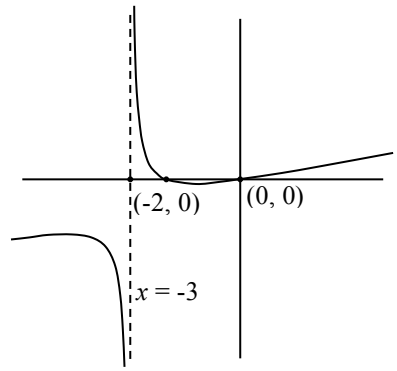
$$\frac{x^2+3x+3}{x+3} \le 1$$

$$\frac{x^2+3x+3}{x+3} - \frac{x+3}{x+3} \le 0$$

$$\frac{x^2+2x}{x+3} \le 0$$

$$\frac{x(x+2)}{x+3} \le 0$$

$$x \in (-\infty,-3) \cup [-2,0]$$



or

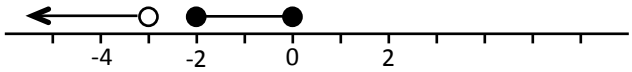
$$\frac{x^2+3x+3}{x+3} - \frac{x+3}{x+3} \le 0$$

$$\frac{x^2+2x}{x+3} \le 0$$

$$\frac{x(x+2)}{x+3} \le 0$$

Critical points where numerator or denominator are zero …
$x = 0$, $x = -2$, , $x = -3$

Test original inequality at each critical point and in each interval …
$x = -4$ is true, $x = -3$ is false, $x = -2.5$ is false, $x = -2$ is true, $x = -1$ is true, $x = 0$ is true, $x = 1$ is false



$x < -3$ or $-2 \le x \le 0$

37. Solve $2x = \sqrt{6-5x}$

$(2x)^2 = 6 - 5x$

$4x^2 + 5x - 6 = 0$

$(4x-3)(x+2) = 0$

$x = \frac{3}{4}, -2$

checking shows $x \neq -2$, so $x = \frac{3}{4}$

38. Solve:

$9^{5-2x} = \dfrac{1}{27^{x-3}}$

$\left(3^2\right)^{5-2x} = \left(3^{-3}\right)^{x-3}$

$3^{10-4x} = 3^{9-3x}$

$10 - 4x = 9 - 3x$

$x = 1$

39. Solve:

$2\log_5 x = 2 + \log_5 3x$

$\log_5 x^2 = \log_5 25 + \log_5 3x$

$\log_5 x^2 = \log_5 75x$

$x^2 = 75x$

$x = 0, 75$

$x \neq 0$

$x = 75$

40. Solve $e^{2x} - 10 = 3e^x$

Let $y = e^x$

$e^{2x} - 3e^x - 10 = 0$

$y^2 - 3y - 10 = 0$

$(y-5)(y+2) = 0$

$y = 5, -2$

$e^x = 5, -2$

but $e^x > 0$

$e^x = 5$

$x = \ln 5$