Opening mathematics texts: resisting the seduction[1]

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Abstract
This analysis of the writing in a grade 7 mathematics textbook distinguishes between closed texts and open texts, which acknowledge multiple possibilities. I use tools that have recently been applied in mathematics contexts, focussing on grammatical features that include personal pronouns, modality, and types of imperatives, as well as on accompanying structural elements such as photographs and the number of possibilities presented. I extend this discussion to show how even texts that appear open can seduce readers into feeling dialogue while actually leading them down a narrow path. This phenomenon points to the normalizing power of curriculum. For this analysis and reflection I draw on mathematics textbook material that I wrote as an alternative to normalizing text. I identify myself as a self-critical author and invite readers to be critical of their reading and writing of mathematics texts.

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"I identified the bases and the height. I noticed that the 6.8 cm side length was extra information that I didn't need" (Small et al., 2008b, p. 145). This excerpt from a mathematics textbook is unlike the text in many mathematics resources because it uses an *I* voice. An *I* voice can show a human making decisions in mathematics, but may also seductively draw textbook users down a normalized path while creating the illusion of decision-making and openness.

This article builds on empirical findings and analysis that have applied appraisal linguistics to identify how mathematics textbooks open and close dialogue (Herbel-Eisenmann & Wagner, 2010; Mesa & Chang, 2008; Wagner & Herbel-Eisenmann, 2008). I begin with background that supports the development of theory — with a definition of *seduction*, an overview of Ellsworth's (1997) work on applying film theory to positioning in education, a description of theories that distinguish between open and closed text, and an account of linguistic tools that have been applied to making these distinctions in oral and written mathematics texts. Next, I apply this analysis to

[1] This article is an elaboration on a paper published in the proceedings of the sixth international conference of Mathematics, Education and Society (Wagner, 2010).

examples from the textbook quoted above. I follow this with a more critical analysis inspired by Ellsworth, moving beyond the tools of appraisal linguistics. The article closes by discussing the inherent tensions between curriculum and seduction and by considering alternative forms of text that could open dialogue by recognizing the choices that people make within mathematics.

For the analysis, I draw examples from a grade 7 mathematics textbook for which I was a co-author. All the examples come from parts of the book I wrote, including the excerpt that opened this article. By criticizing my own writing, I stand with other mathematics textbook writers facing the tensions of making choices in presenting mathematics. Furthermore, as I will develop in the conclusion, presenting myself as a self-critical author exemplifies an alternative to traditional mathematics textbook voice, albeit in a different context.

The *I* voice in the textbook series was one of several planned features aimed at supporting student understanding and communication experience within mathematics. This choice of voice was instrumental in my willingness to participate in the writing because in my research I had focused on identifying human particularities and choices in mathematics. Although the *I* voice seemed to fit my vision for mathematics textbooks, I was not completely satisfied when I used the *I* voice in my writing, as I will develop below. Ellsworth's (1997) investigation of the way teachers address students challenged me to reflect on the seduction at work in mathematics textbooks.

1 Positioning for seduction

1.1 Leading away/astray

Seduction is related to attraction, often with an element of intent; the seductive person tries to make himself/herself or his/her ideas attractive to others. There is a sense of manipulation; the person is seduced to be attracted to someone or something to which she or he would not normally be attracted. The etymology of the word supports these conventional, current associations. The Latin *sēdūcere* means to lead away: *sē* means *away* or *aside* and *dūcere* means *lead*. The seduced person is usually taken as passive — we say the person is led, not that the person chooses to follow.

For centuries, the Latin *sēdūcere* had the same sense of leading astray that continues with the English *seduce* (Barnhart, 1988, p. 979). The idea of leading astray assumes a right path. But recent examples of colonialism problematize the idea of a right path. When people have thought they were drawing others to a right path, future generations have instead judged them for leading these others to destruction. Thus I challenge the negative sense associated with seduction, preferring to think of it as leading *away* rather than leading *astray*.

All text leads readers away while also drawing them in. A common thread in Barthes's (1975) demonstrations of text drawing in readers is the reader's choice to accept the text as being for him or her: "The text you write must prove to me *that it desires me*. This proof exists: it is writing" (p. 6). When I read, I take the text (and perhaps the author) to be addressing me, just as when I listen to someone I take them to be addressing me, responding to their sense of my needs. However, the relationship between reader and author, and between speaker and listener, is not so straightforward. Bakhtin (1975/81) described how utterances simultaneously draw in and lead away — when I address someone I appeal to shared meaning (drawing my listener/reader in) in order to bring new meaning (leading my listener/reader away). In Section 1.3 I elaborate on the theorization of this tension.

1.2 Addressing the reader

Ellsworth (1997) used a theoretical lens informed by critical film studies to describe the positioning in typical classrooms. She drew attention to modes of address in film and in classrooms. The operative question for her is, “Who does this film think you are?” (p. 22). In my context, the related questions are, “Who does this teacher think you are as a student?” and, more relevant to textbook writing, “Who does this textbook think you are?”

Ellsworth’s question relates to Eco’s accounts of the model reader. Eco (1994) described how texts create a “model reader — a sort of ideal type whom the text not only foresees as a collaborator but also tries to create” (p. 9). In this sense, I think of a seductive text saying, “I know who you are. I am giving you what you want.” The text addresses the needs of a real reader enough to transform him or her into the reader imagined by the text. I say that it is the text, and not the author, that imagines and addresses the reader because a text constructs a model reader regardless of the author’s intent.[2]

1.3 Opening and closing dialogue

When Eco (1979) developed the idea of a model reader he noted that there are different kinds of model readers. A *closed text* imagines and constructs a single reader. Only one interpretation is recognized. By contrast, in an *open text* “the author offers […] the addressee a work *to be completed*. [The author] does not know the exact fashion in which his work will be concluded, but he is aware that once completed the work in question will still be his own” (p. 62). The text invites the reader to choose from a variety of interpretations. Weiss (2010) has applied Eco’s distinction to evaluate the way teaching was represented in some animated vignettes of mathematics teaching episodes.

Eco’s sense of closed and open texts relates to the orienting distinction made in appraisal linguistics — linguistic resources can be “broadly divided into those which entertain or open up the space for dialogic alternatives and, alternatively, those which suppress or close down the space for such alternation” (White, 2003, p. 259). White connected texts that open dialogue to Bakhtin’s (1975/1981) notion of heteroglossic interaction. And he connected texts that close dialogue to the notion of monoglossic utterances (see also Martin & White, 2005). Appraisal linguistics uses criteria similar to Eco’s to analyze lexical and grammatical aspects of text to identify whether the text is opening or closing dialogue. If the grammar recognizes multiple points of view, the text is appraised to be open. Eco looked for acknowledgment of diversity among potential readers, but in appraisal linguistics diversity is recognized in multiple ways. For example, when speakers or authors refer to their choices, they recognize the potential for multiple points of view.

Though appraisal linguistics theorizes its distinction between open and closed texts using Bakhtin, he did not focus his notion of heteroglossia and unitary (monoglossic)

[2] Eco (1979) theorizes the distinction between the intent of a work and the intent of its author.

language on evaluating texts as being open or closed. Rather, he was pointing to phenomena present in all language. Unitary language acts like centripetal force, pulling meaning to a unified centre, while heteroglossia acts like centrifugal force, pushing out from shared meaning to say something new. Both forces are always present: "Every concrete utterance of a speaking subject serves as a point where centrifugal as well as centripetal forces are brought to bear" (p. 272). Barwell (2011) noted the presence of both forces in tensions he identified in multilingual mathematics classrooms. For my reflections in this article, I follow the appraisal linguistics tradition of evaluating features of text that open or close dialogue, though I am aware that any text carries both forces. The question for me is which force dominates.

Positioning theory reminds us that one can always resist the positioning initiated in any interaction (van Langenhove & Harré, 1999); readers can comply with a text that discourages dialogue or they can resist, either by following the unitary line or reasoning or by raising alternative ideas. Likewise, readers can comply with or resist texts that open dialogue. As Bakhtin reminds, the fact that the text is given (or spoken) to someone is a tacit invitation for engagement. A text can open or close dialogue by reminding the reader that alternative views are possible or by obscuring this possibility, but text cannot close dialogue off completely. Indeed, any text is part of a dialogue.

When Rotman (1988) analyzed semiotics in mathematics, he noted a distinction between imperatives. This distinction also relates to open and closed texts. He distinguished between inclusive commands, which imagine more than one person, and exclusive commands that are pointed at one person. He called exclusive imperatives (such as *write* or *put*) "scribbler" commands (p. 10) because the reader is expected simply to follow directions. He called inclusive imperatives (such as *explain* or *prove*) "thinker" commands (p. 10) because the reader is expected to reflect on and interact with a world. However, Rotman noted that mathematical work is often done in isolation. Thus one person can be both a scribbler and a thinker— a split subject. The thinker imagines worlds and the scribbler is the agent of the thinker, acting in these worlds. In such internal dialogue, Rotman placed the person, for whom an *I* voice is available, in a triadic relationship with the scribbler and thinker.

Ellsworth (1979) described how camera work in film has effects similar to texts that close and open dialogue. Like Eco, but in the context of film instead of written texts, Ellsworth made clear that there is a difference between a real viewer and the viewer constructed by a film. "Multiple entry" (p. 27) refers to a film's connection to a diverse audience. It is a necessity in the film industry because commercial viability depends on it. Ellsworth used the example of *Flashdance*, which appeals to adolescent girls and boys for different reasons. However, even while appealing to diverse needs, a director can arouse the viewer's "empathy for and imaginative collusion with a character's intentions, experiences, goals" (p. 30) by, for example, filming shots from the principal character's point of view. Thus a film may address multiple points of view even as it privileges one.

In alternative cinema, Ellsworth said directors often break typical forms to avoid seducing audiences. The seduction happens when one point of view is promoted as meriting viewer empathy and collusion. "The revolutionary hope was that changing modes of address in films might change the kinds of subject positions that are available and valued in society" (p. 30). "While audiences can't simply be placed by a mode of address, modes of address do offer seductive encouragements and rewards for assuming those positions within gender, social status, race, nationality, attitude, taste, style, to which a film is addressed" (p. 28). This clarified for me that mathematics textbooks might seduce readers in similar ways to assume certain positions. My hope for mathematics, like the hope Ellsworth identified in alternative film, is that alternative forms of address, particularly in mathematics textbooks, may encourage openness to multiple points of view.

When a mathematics textbook addresses students in a way that closes dialogue, it privileges a particular point of view by suggesting that this point of view is normal. The effects of such normalization can be dangerous, even if the point of view has some value. Foucault (1975/1977) described in detail some of the negative effects of such normalization. The power of the Norm, according to Foucault, imposes homogeneity by underpinning "classification, hierarchization and the distribution of rank" (p. 184). The Norm pervades society in many institutions. The problem he described is not that the characteristics taken to be normative are necessarily bad, but rather that society is pushed toward a dangerous uniformity.

It is important for mathematics educators to reflect on the value and dangers of normalization in mathematics learning contexts. I have described my sense of the normalization that is particular to mathematics in my contribution to a collection of essays by members of the Nonkilling Science and Technology Research Committee, which advises the Center for Global Nonkilling (Wagner, 2011), but I will not elaborate much on this here. Rather, I will take it as a given that normalization has dangers. In contrast to closed text, open text that acknowledges or even celebrates the value of multiple perspectives can promote an understanding of mathematics in action. The history of mathematics is rich with examples of exploring alternatives to apparently normal ways of representing the world — for example, the invention of imaginary numbers. Most importantly, closed texts are associated with authoritarian positioning, in which something is taken to be true not on its own merits but because one trusts the source and does not question the normalized view.

1.4 Features of mathematics textbooks

Before writing for the textbook series discussed in this article, I had done critical analysis of texts, which gave me a sense of some things to avoid in my writing. In addition to building from contexts that would be meaningful to the readers and writing in a way that directs students to understand key mathematical concepts and procedures in the curriculum, I attended to grammatical features that had figured in my textbook criticism. Notably, grammatical and lexical choices in a textbook position students in relation to each other, to mathematics, to their teacher, and even to others in their larger context.

In critical analysis of two middle-school mathematics textbooks, Herbel-Eisenmann and I drew attention to personal pronouns (Herbel-Eisenmann & Wagner, 2007). We noted that these volumes contained few personal pronouns and no first person singular pronouns; there were some instances of *you* and *we* but no instances of *I* or *me*. Morgan (1998) has shown that the absence of first person personal pronouns (*I*, *me*, *my*, *we*, *us*, *our*) masks human agency in mathematics. I add that the same is true for second person pronouns (*you*, *your*). In oral discourse in mathematics classrooms, teachers and students become aware of human agency in mathematics as they begin to reference human subjects in the discourse practice (Herbel-Eisenmann & Wagner, 2010).

When a text shows people making choices, the reader sees that she or he too can make choices in mathematics. Thus the text is open or heteroglossic. In English-medium classrooms, the absence of *I* and *me* is even more powerful in obscuring human agency than the absence of *you*, *we*, and their related pronouns because in English *you* and *we* can be used in a generalizing sense. Rowland (2000) has described how in English contexts the pronoun *you* can mark generalizations in mathematics by referring not to anyone in particular but to everyone in general. Thus mathematics texts that have human subjects in their sentences show humans making choices, but the common sense of a language may obscure the human subject by implying a generalized practice.

There are other ways of using language to acknowledge the possibility of multiple points of view. Modality refers to the range of certainty expressed in speaking or writing. One way to express degrees of certainty is to use modal verbs. For example, "it *must* have two parallel sides" is stronger than "it *could* have two parallel sides." *Must* and *could* are modal verbs. Expressions with more certainty have high modality and those with low certainty have low modality. Bald assertions, which Rowland (2000) has called *root modality*, are stronger yet than expressions involving the highest modal verbs. Using the declarative mood to say, "it has two parallel sides" does not recognize any alternatives. By contrast, a modal verb, even if it is a strong one like *must*, suggests that others may disagree — "it *could* have two parallel sides" or "it *must* have two parallel sides." The declarative mood often presents information without a human subject, using no personal pronouns. Mathematics texts that index uncertainty — by using modal verbs, for example — show that humans make choices in mathematics.

2 Identifying open and closed text

In this section I will draw on textbook material that I wrote in order to make distinctions between texts that open and close dialogue. I will identify ways in which text may be seen to seduce by leading toward a particular closed point of view. And I will discuss how text might open dialogue by recognizing or promoting multiple points of view. This analysis will follow the kind of analysis referenced in the previous section. In the next section, I will apply a more critical analysis inspired by Ellsworth (1997).

Though I contributed to eight books in the *Understanding Mathematics* series (four student books and their associated teacher's guides), I will focus here on excerpts from the Grade 7 student book (Small et al., 2008b), which I chose because it includes examples of the full variety of textual forms I want to discuss. All the excerpts are from chapters I wrote.

This textbook series was commissioned by the Ministry of Education in Bhutan to be a classroom resource; each student would have his or her own textbook. The accompanying teacher's guides tell teachers how to use the textbooks, lesson by lesson. The series was to follow the outcomes prescribed in the new curriculum recently introduced by the Ministry to replace the previous regime, which followed Indian curriculum. India is Bhutan's primary neighbour and trading partner, so it is sensible to have curriculum that prepares students for tertiary education in India. However, educational leaders in Bhutan sought to develop a uniquely Bhutanese curriculum that addressed Bhutanese contexts and aligned with international foci.

When the author team developed the *Understanding Mathematics* series, we agreed that we would include an *I* voice in most of the lessons. This choice fit well with my views on recognizing human choice in mathematics. The general structure of a lesson was a quick exploration called *Try This*, an *Exposition*, a return to the *Try This*, some *Examples*, and a series of questions called *Practising and Applying*. Each unit in the textbook series also included an investigation lesson, games, a performance task, and sections on mathematics in culture. My analysis here focuses on the regular lessons. For this I use examples from each of the parts of the regular lesson to consider how the text opens and closes dialogue.

2.1 Examining the opening — *Try This*

Each *Try This* addresses the reader directly with imperatives and questions. This section opens each lesson, but opening a lesson does not necessarily require the opening of dialogue. Nevertheless, the author team agreed that we wanted teachers to open dialogue by inviting students to do something — if each student has his or her own experience from this action at the outset of a lesson, together they could have multiple points of view for discussion. The front matter of the teacher's guide addresses teachers directly: "The reason to start with a *Try This* is that we believe students should do some mathematics independently before you intervene" (Small et al., 2008a, p. xiii). Thus there is a sense of student action, but even with action the space allowed for decisions can vary. For example, in a lesson called "Area of a Trapezoid" the *Try This* shows a trapezoid drawn on dot paper, as shown in Figure 1. The instructions are:

> This polygon is drawn on 1 cm dot paper.
> A. i) Find its area by dividing it into a rectangle and two triangles.
> ii) Find its area by dividing it into two triangles.
> iii) Show another way you can divide the polygon into two triangles.
> (Small et al., 2008b, p. 144)

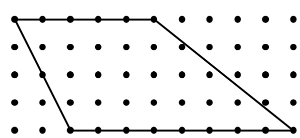

Figure 1 Trapezoid on dot paper in *Try This*

Using Rotman's (1988) distinction between inclusive thinker imperatives and exclusive scribbler imperatives, this *Try This* begins with scribbler imperatives (*find* and *divide*) and works toward a thinker imperative (*show*), at least nominally. *Show* is a thinker imperative because it implies interaction; the reader is supposed to show something to someone else. But, as discussed by Rotman, the interaction characteristic of a thinker imperative may be internal to an individual. Furthermore, actual practice in the mathematics classroom may muddy the sense of such imperatives, perhaps transforming a thinker imperative into a scribbler imperative when the teacher does not direct students to share their results. The same goes for the verb *explain,* another thinker imperative that appears often in mathematics textbooks. Textbooks or teachers can promote the inclusivity of these thinker imperatives by directing students to show or explain their ideas to each other. This is encouraged in the teacher's guides.

Despite the apparent openness of part iii, there is only one way of performing part i and there are only two ways of performing part ii. These are shown in Figure 2. Because a student in isolation can find the required divisions of shapes, these are scribbler imperatives. For part iii, the student is expected to show another possibility to someone else (perhaps a peer), or perhaps to convince him- or herself that there is at least one other possibility (the split subject in dialogue), so it seems to have a thinker imperative. However, because there is only one remaining possibility for dividing the polygon (whichever way was not chosen for part ii) there is little opportunity to think differently. Though the structure of working from independent scribbling to interactive thinking is sound (in my view), it still leads the model reader down a narrow path. Thus the text closes dialogue.

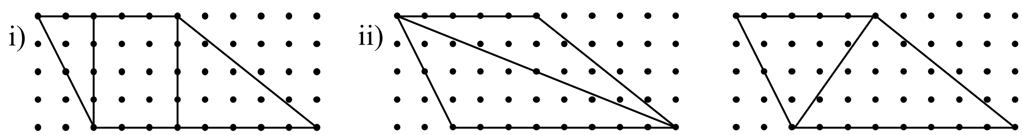

Figure 2 Limited ways of performing the *Try This*

If the author's voice were recognized in the text (and not hidden behind imperatives), this *Try This* might appear more open because the reader would see that someone had to decide which instruction to give. But if the text said, "I would like you to divide the polygon into two triangles," the reader might wonder why the author chose to give this direction. This kind of wondering may be either productive or distracting. In oral mathematics classroom interaction, it is common for a teacher to direct activity with such requests that are based on personal authority relationships (Herbel-Eisenmann, Wagner & Cortes, 2010). One problem with this construction is that it positions the student as passive and reliant on the teacher as authority. The student performs the prescribed mathematics to please the teacher or to follow the textbook rather than to address his or her own mathematical reasoning needs. Nevertheless, when the author is masked it is harder to question the text. This text does not even pretend to follow the reader's reasoning.

2.2 Examining the *Exposition*

The *Exposition* is more closed than the *Try This*, as it comprises a series of assertions about what trapezoids are and what the formula is. It begins like this:

> A trapezoid is a quadrilateral with exactly two parallel sides.
> a and b represent the two bases — the parallel sides.
> h represents the height, which is perpendicular to the bases.
> In an isosceles trapezoid, the sides that are not parallel are equal in length.
> The formula for the area of a trapezoid is: Area of a trapezoid = $(a + b) \times h \div 2$
> (Small et al., 2008b, p. 144)

The textbook does not show a person naming the shape or developing the formula; the shape and formula simply exist. All the sentences are in the declarative mood. Human decision-making could have been highlighted by identifying whomever chose the word 'trapezoid' to describe the shape. A problem for authors wishing to do this is that it is often impossible to identify the originator of a term. To draw attention to the complexities of how a term has come to be taken as shared would draw attention away from the prescribed task at hand, which is to think about the characteristics of the shape. Discussion about the evolution of vocabulary could highlight the human choices behind mathematical development. This complexity is especially interesting in this case because the Greek *trápezoeidés* means table-shaped (Barnhart, 1988, p. 1162) so the question is when to raise this point.

Similarly, the textbook does not show a person developing the formula. Up to this point in the *Exposition*, it would appear that the formula predates humanity. To mitigate this, I continued the *Exposition* with an attempt to draw in the student by using a *you* voice, asking the reader to imagine how two congruent trapezoids fit together. With this thought experiment, I directed the reader to manipulate and think about the pair of trapezoids to develop the given formula (the text included a diagram to help with visualization):

> The formula makes sense if you think of a parallelogram as two congruent trapezoids:
> Rotate a trapezoid 180° around the midpoint of one of its non-parallel sides.
> The two congruent trapezoids together make a parallelogram.
> The height of the parallelogram is h and its base is $a + b$.
> The area of a parallelogram is the length of the base times the height.
> For this parallelogram, it is $A = (a + b) \times h$.
> Since the parallelogram is made up of two congruent trapezoids, one of the trapezoids is half the parallelogram. Its area is $A = (a + b) \times h \div 2$.
> (Small et al., 2008b, p. 144)

These scribbler commands guide the student to sketch and visualize the shape in a particular way. They could be seen as a response to the reader's implied question: "Show me how the formula makes sense." But there is no real interaction and I presented only one way of manipulating and visualizing the trapezoid to understand the formula even though other ways are possible. This text still closes dialogue.

2.3 Examining the *Examples*

For me, the most interesting part of the grammatical structuring of the mathematics occurs in the *Examples* that follow the *Exposition*. Here the other authors and I used the *I* voice. I do not think having a closed text up to this point would be so problematic if there were movement toward an open text because the part of the text that is open would invite multiple points of view and could turn the earlier closed text into what White (2003) called a retrospectively dialogic text; once a reader's attention is drawn to alternative possibility, even a previous text that was structured to be closed becomes open for question or examination.

The *Examples* in this textbook series were structured such that a question is followed by a two-column table. The left-hand column shows a *Solution* and the right-hand column shows the related *Thinking*. In the top right corner of each *Thinking* section there is a photograph of a Bhutanese child who appears to be doing mathematics. There was a bank of six different photos to draw on, three girls and three boys. In each photograph the child is looking down at an open notebook and holding a pencil up to his or her right temple. (The one exception shows a boy holding the pencil in his mouth.) Each child is supposed to embody thinking.[3] Some *Examples* have more than one *Solution* and associated *Thinking*, each with a different student photograph.

Because I saw these sections as an opportunity to use open text, I was especially aware of the positioning I initiated. However, I struggled to exemplify some curriculum outcomes with open text. The outcome "apply the formula for the area of a trapezoid" is by nature a scribbling task using Rotman's (1988) distinction, because *apply* is an exclusive imperative. Thus the *Thinking* for the one *Solution* to an *Example* in this lesson does not open up alternative possibilities.

Given the task, "Determine the area of this trapezoid" (Small et al., 2008b, p. 145) along with a diagram, the *Thinking* goes like this:

> I knew it was a trapezoid because the arrow marks showed that it had exactly two parallel sides.
> I identified the bases and the height. I noticed that the 6.8 cm side length was extra information that I didn't need.
> I used the formula.
> (Small et al., 2008b, p. 145)

All the verbs are exclusive, scribbler verbs because the student can do this in isolation. He or she 'knows,' 'identifies,' 'notices,' and 'uses.'

Other outcomes were easier to write as open texts. For example, for determining the area of composite shapes, I provided two *Solutions* for one of the *Examples*. Using two solutions was encouraged among the authors because the two voices would demonstrate that multiple solutions could be possible in mathematics. In one *Solution* my fictional student used addition, dividing the given polygon into a rectangle and three triangles. In the other *Solution* my fictional student used subtraction, drawing a rectangle around the shape and identifying the triangles outside the polygon. By showing two methods I meant to suggest that multiple approaches are possible, but a reader could also assume that there are exactly two ways of performing the task (as was the case in the *Try This* discussed in Section 2.1). Again, the relevant curriculum outcome is performative; the outcomes are structured as imperatives, many of which are scribbler imperatives. In this case the outcome reads, "estimate and calculate the area of shapes on grids."

I found it easier to use open text featuring thinker imperatives to develop outcomes that required understanding or generalization. For example, to address the outcome "determine if certain combinations of [triangle] classifications can exist at the same time"

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[3] There was significant discussion among the Bhutanese educators who commissioned this textbook series regarding the nature of these model student images: should they be cartoon-like caricatures or actual photographs? I do not understand sufficiently the cultural implications of their discussion and decision, but I have noted implications for the representation of humans as subjects of mathematics. A cartoon image is relatively generic and may reflect the penchant for generality in mathematics and the related obfuscation of human particularities. Dowling (1988) also noted distinctions between cartoon images and photographs in his analysis of mathematics textbooks; the intertextuality between such images and other texts read by students can position the students in various ways.

my *Example* asked, "Is it possible for a right triangle to also be isosceles? How do you know?" (Small et al., 2008b, p. 112). The respondent has significant latitude. The *Thinking* I wrote went like this:

> • Before I tried to draw it, I thought about whether it was possible.
> • I knew an isosceles triangle had two equal angles and a right triangle had a 90° angle.
> • I also knew the sum of the angles of a triangle was 180°.
> • So, in a right isosceles triangle, there had to be a 90° angle and two 45° angles.
> • I sketched the triangle and it looked possible. My sketch also helped me draw the triangle.
> (Small et al., 2008b, p. 112)

The human subject at work is demonstrated by the *I* voice, but much of the *Thinking* employs root modality, using declarative mood sentences prefaced with "I knew." Some of the statements use the modal verb *had to*, but the verb does not suggest that a human is compelling or requesting things. Reason is the compelling force. Rowland (2000) called this kind of modality *alethic modality* because it is driven by logic, unlike *deontic modality*, which indexes human obligation. As with the possibly implied student question in the *Exposition*, the "How do you know?" question seems to demand a response, but there is no human to hear the response. Thus the model response cannot bear features that would be present in human interaction; the dialogue must be internal. In this way the parallel *Thinking* and *Solution* sections embody Rotman's split subject, where the *Solution* is the scribbling half of the scribbler/thinker duo. The text is relatively open because of the modality and the question that suggests interaction, even if the interaction is between the scribbler and thinker within an individual student.

I had resigned myself to writing mostly closed text for scribbler curriculum outcomes. But relatively open *Examples* like this one became interesting to me from the perspective of appraising the way text can seduce a reader. As I elaborate in the next section, the *Examples* model responses in a way that suggests interaction though there is actually no interaction because the text is static.

3 Identifying seduction

Ellsworth (1997) pointed out that realist representations can develop and maintain an illusion of difference. She focused on the illusion of dialogue in classrooms and claimed that dialogic forms of interaction can mask an undercurrent of control. With this illusion, understanding is the proclaimed goal. Conscious intention and consensus are valued while desire, conflict, and ambiguity are scorned:

> By presenting themselves as *desiring only understanding*, educational texts address students as if the texts were from no one, with no desire to place their readers in any position except that of neutral, benign, general, generic understanding. (p. 47, emphasis mine)

Is something similar at work in the *Understanding Mathematics* textbook? The book is explicitly oriented to develop understanding; that is clear in its title. Do the dialogic or open forms of text, such as the multiple *I* voices in the *Examples* and the thinker imperatives in other parts of the text, work as an illusion? In other words, does the text seduce the reader by suggesting open dialogue while maintaining closed positioning?

In the *Thinking* about isosceles right triangles shown above, which was the most open of the texts discussed here, the model student listed things he knew and he drew a sketch (the picture for this one is a boy). His sketch included markings that showed the things he knew. I promoted these actions because I saw them as effective strategies to help students develop an understanding of the parameters and then form a generalization related to the parameters. It seems to me that educators are usually expected to promote effective strategies in this way.

The normalizing *I* voice is similar to the seductive camera technique Ellsworth (1997) described. Writing from the point of view of a model student is like positioning a camera from the point of view of the protagonist in a film. The seduction comes from the ease with which readers or viewers can see themselves in the place of the model student or protagonist as the alignment is not interrupted in any way.

The reader can easily see him- or herself in the place of the model student, who is represented with a photograph of someone the same age and nationality dressed in a school uniform like his or her own, and who appears alongside an *I* voice. The reader is seduced into writing and thinking like the model, seduced into becoming the model reader.

This kind of writing (and thinking) that reflects the model reader bears similarities to a child recognizing him- or herself in a mirror, as described by Lacan (1966/2002), except that it is an opposite. With reflections it is easy to mistake the image for the original, and thus the recognition of similarity underwrites the seduction. Instead of the possibility for transformation that accompanies writing from one's own experience, which has been described by de Freitas & Paton (2009), transformation is repressed by a seduction to conformity where the child's writing reflects the experience of some non-present ideal student.

Reflecting on these observations about open, closed, and seductive text in this mathematics textbook highlights the underlying structuring power of official curriculum. If I write a textbook that 'follows' the official curriculum, which is the norm for textbook structuring, then it seems inevitable that I will create closed texts if the curriculum has performance-based outcomes. Scribbler imperatives in closed texts demand that students perform narrowly defined procedures. Thinker imperatives may also be deemed performance-based outcomes, but they are different. They seem to invite more latitude and so they give writers of curriculum-following textbooks more options for structuring text. Though in my earlier analysis of mathematics textbooks I decried the lack of an *I* voice, my experience of trying to write with an *I* voice leads me to recognize that there are subtle dangers in using it. An *I* voice may seduce students who read the text to take up the one point of view presented to them despite the author's intention to have it represent the possibility of multiple points of view. The relatively open text with its multiple points of view gives the sense that the student is making choices and moving a dialogue forward. However, this is an illusion because the curriculum is already laid before the students. The path is pre-defined.

4 Discussion

Before reflecting on the way mathematics texts may open dialogue more authentically, it is important to consider briefly the context of the texts I used for my analysis. Any text crosses certain levels of culture, but the *Understanding Mathematics* series is a product of interactions that span the globe.

My engagement with educators in Bhutan followed in the footsteps of many years of cooperation between my university and the Ministry of Education in Bhutan. The Bhutanese government strongly directs its international partnerships by writing needs assessments based on its identification of long-term goals and choosing to work only with trusted partners. Nevertheless, this strong leadership does not completely avert the complexities of interpersonal and intercultural positioning in international partnerships.

I was in Canada when I wrote for the *Understanding Mathematics* series, though I had worked both in Bhutan and with Bhutanese students in Canada. When the writing was complete, I co-facilitated a writers' workshop in Bhutan in which Bhutanese mathematics teachers helped the writing team revise drafts of the books to ensure the accuracy of cultural contexts, to suggest further useful contexts, to give insight into local teachers' and students' readiness for understanding the material, and to suggest alternative text that would better fit teachers' and students' needs. This approach was stipulated by the Ministry of Education, which determined that Bhutan did not yet have the experts to write the textbooks. Indeed, this was the first step away from a dependency on India. The structure of the partnership was carefully considered with explicit wariness among both Bhutanese and Canadian partners of colonialist relationships. Even so, the structure was not above critique.

Similar dynamics would be at work in the development of any mathematics textbook, though the cultural differences may not be as obvious and the distance between authors and readers may not be as extreme.

4.1 Normalization and privilege in 'curriculum for all'

My sense is that the closed and sometimes seductive nature of typical text in mathematics textbooks, and in particular the text that I wrote, is connected to other seductions that relate to human difference and diversity in interaction. Thus I ask what other needs I am meeting through my writing and how fulfilling these needs relates to the seduction or narrowness of mathematics I present.

I have a personal need to be relevant. It feels good to be relevant in developing mathematics education beyond my country's borders. The same dynamic would be present even for textbooks written by nationals within a country but I will write here about my own experience. Far-reaching connections enhance my reputation as an educator. Similarly, connecting to outside experts lends authority to Bhutanese educators. How do these needs connect with the text I produced? For the teachers who mediate the text in their mathematics classrooms, we (the textbook writers and Bhutanese leaders) lend them expertise and authority so that they too can be relevant. And part of the explicit purpose of education is to equip students to be relevant to society.

With all this valuing of personal relevance, I have to ask what relevance is. It seems to be closely related to particular values, though the word itself appears values-free. In this way, the goal of relevance is seductive. Relevance in general seems like an unquestionable need but any particular act of trying to be relevant would index a value set that is masked by the grammar and lexicon of objectivity. For example, my act of 'helping' Bhutanese educators revise their curriculum suggests both that the curriculum needs revision and that I know what Bhutanese students need. So while it seems that the focus is on Bhutan's culture, in fact my non-Bhutanese culture and experiences are being privileged in various ways. The supposed superiority of my culture is reinforced by the idea that Bhutanese educators need my kind of help to foreground their own culture. This conundrum relates to Dowling's (1998) critique of ethnomathematics, which he called the "myth of emancipation" (p. 11). It also relates to the ever-present tension between heteroglossic and unitary text identified by Bakhtin (1975/1981) — all text (any interaction) borders on privileging a point of view while opening points of view.

I need to experience other cultures — other points of view — in order to better understand the people in the world around me, both locally and globally. This need draws me to travel and work alongside people in Bhutan and elsewhere. It relates to my understanding of the benefit of helping students to value multiple points of view. Paradoxically, it is as I interact across cultures that I better understand the problems of intercultural interaction. Thus the alternatives seem to be either naïve lack of engagement, naïve engagement, or acceptance of interaction that will have its problems.

I bring to my textbook writing (and other interactions) my views, which are informed by the culture(s) I inherit as well as by the culture(s) in which I have chosen to immerse myself. I cannot avoid privileging these views of mine to some extent. The operative question then is the extent to which I acknowledge diversity of points of view — to what extent do I open dialogue with my texts? I found it challenging to write text that provides readers with the experience of diverse points of view while also following a curriculum that aims for particular kinds of performance.

Just as reflecting on the representation of curriculum outcomes in mathematics text exposed the role of curriculum in closing texts, reflections on the larger context of applying curriculum across contexts exposes the role of curriculum in structuring a closed mathematics. Brown et al. (2007) have explained how standards in curriculum infer an ideology like colonialism by mandating outcomes for 'all.' A mandated curriculum for all promotes a particular norm for each individual student, regardless of the person's goals or cultural context.

Returning to the conception of seduction as a leading away from the right path, it is instructive to note that the word *curriculum* in Latin means path, referring to the path someone runs (Barnhart, 1988, p. 244). Standards-based curricula are designed to lay a path before students, 'the right path.' The expectation that teaching is required to mediate students' movement to and along this right path is a form of leading. Teachers and school texts lead students away, seducing them from whatever path they would have otherwise been on. I suggest that the assumption that curriculum outcomes define a right path implies that children are otherwise on a wrong path (perhaps like the concept of original sin).

4.2 Alternatives

The difficult experience of writing mathematics texts for school in a dialogically open way compels me to consider alternatives. It seems that the central problem to structuring an open text is the normalizing power of official curriculum. The problem is that the right path is predetermined.

I see three ways of overcoming this power. One is to ignore it, as some innovative writers do. Stocker's (2006) *Maththatmatters: A teacher resource linking math and social justice* is a collection of lessons that centre on social injustices and direct students to do mathematics that helps them understand the injustices in a particular way. As much as I approve of Stocker's attention to social justice concerns that I share, I recognize that ignoring curriculum would be difficult for a teacher who is required by law to 'deliver' certain curriculum. Stocker's book would have to be supplemented by other resources. Frankenstein (e.g., 1989) and Gutstein (e.g., Gutstein & Peterson, 2005) have also developed innovative practices and texts that, like Stocker's, aim to address social injustices and would require supplementation in contexts where curriculum is prescribed externally. In short, I see Frankenstein, Gutstein, and Stocker promoting radically different curriculum that foregrounds social issues and positions mathematics as a tool for addressing these issues.

This suggests a second way to counter the normalizing force that dominates mathematics learning: change the nature of prescribed outcomes in mathematics curriculum. School authorities could require mathematics classrooms to be sites of critiquing mathematics in social action even while doing mathematics. Students could use mathematics to interrogate their world and to interrogate the way mathematics is used in rhetoric in society. This kind of curriculum, as promoted by Frankenstein, Gutstein, and Stocker, could become mainstream. Nevertheless, as long as curriculum prescribes procedures and not critique it is necessary to consider other possibilities for mathematics educators.

A third way to overcome the normalizing force of curriculum is to challenge it in the text resources used by students. This could be done even in a textbook that follows a prescribed and closed curriculum. This approach, which would resemble writing under erasure as described by Derrida (1976), allows for the possibility of presenting the curriculum while at the same time questioning it. This would require an authentic *I* voice — an author who reveals him- or herself to be reflecting on the things society expects of students. It would require the writer to reveal values outside mathematics itself as a stance from which to think critically about mathematics and the mathematics curriculum prescribed by the school jurisdiction authorities. Social justice concerns, such as those raised by Frankenstein, Gutstein, and Stocker might provide an appropriate orientation.

There are various ways of introducing such an authentic *I* voice. Fauvel (1989) described how Robert Record, who wrote the first English-language mathematics textbooks, used Platonic dialogue. In this form, errors figured prominently as opportunities to explain both technical details about mathematical procedures and philosophical considerations, such as the appropriateness of using mathematics in a given situation. For example, in one instance the Master tells the Scholar, "here are you twice deceived. First, in going about to add together two sums of sundry things, which you ought not to do, [… and secondly] in writing 14, which came of 6 and 8" (from *The Ground of Arts* (1543), as quoted in Fauvel, 1989, p. 4). In addition to critiquing both a procedure and the appropriateness of that procedure, this approach reveals the messiness and variety of possible approaches within mathematics. For example: "This procedure leaves you with a remainder, so how should this be expressed most accurately? 'There be as many ways', says the master, 'as there be writers almost'" (Fauvel, 1989, p. 5).
An *I* voice alone is not sufficient. There has to be a critic.

Teachers mediate textbooks in their classrooms, as described by Herbel-Eisenmann (2009). From this position teachers can raise the necessary critical voice. Love & Pimm (1996) in their discussion about ways of looking at mathematics textbooks identified the inherent authority of such text. They reminded teachers and students that together "their responses to it may range from taking it for granted to seeing their role as challenging and criticizing it (to interrogate and even deconstruct the text)" (p. 380). However, teachers, like students, are readers of the textbooks they use in mathematics classrooms and are likewise susceptible to being seduced by the text. This is why it is important that the texts themselves avoid seduction by including self-critique. Pimm (2009) addressed this dynamic when commenting on accounts of teachers using textbooks:

> For myself, materials and texts are at best seen as one starting point; they usually require teachers to be thoughtful, aware, and autonomous to use them successfully. To what extent do the texts themselves help to bring this state of affairs about? (p. 196)

Self-critique is also central to the way I am writing this article. I have chosen an *I* voice. I present analysis not in a detached way that implies objectivity, but rather as self-critical reflection. In this way, this article's text is open to dispute and to different interpretations. I am vulnerable. I raise sore points and challenges. For example, I use appraisal linguistics to evaluate text as opening or closing dialogue while I acknowledge that all text does a little of both. I wrote textbooks for Bhutan though I have critical questions about the nature of the relationships involved. And I wrote textbooks with awareness of the tensions in writing text that opens rather than closes. The disclosure of these tensions may undermine my authority because of the moral complexities of writing pedagogical text. But disclosure may also substantiate my authority as I position myself as a self-aware author — "A final snare: renouncing any will-to-possess, I exalt and enchant myself by the 'good image' I shall present of myself" (Barthes, 1978, p. 233). This same dynamic could be present in a mathematics text that critiques itself. It may question the authority of the discipline and the authority of societal decision-makers and their prescribed values and outcomes. In so doing the text would present itself as a critical authority and invite students to take up that same position of authority.

Such radical departures from traditional mathematics textbooks may, however, be rejected by their readers, just as much alternative cinema has been ignored by the masses. Perhaps the absence of such examples of radical texts reflects society's lack of desire for them. A film theorist raised this concern about alternative cinema: "Unless a revolution is desired […] it will never take place" (Wollen, 1982, p. 88). If mathematics students reject alternative texts and embrace closed texts then we will need to ask why students desire closed texts. What is so uncomfortable about the alternatives? But it is still necessary to explore the possibilities of mathematics texts written by a critical self-identifying author.

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