Investigation, mathematics education and genre: an essay review of Candia Morgan's *Writing Mathematically: the Discourse of Investigation* [1]

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> the viability of a genre like the viability of a family is based on survival, and the indispensable property of a surviving family is a continuing ability to take in new members who bring fresh genetic material into the old reservoir. So the viability of a genre may depend fairly heavily on an avant-garde activity that has often been seen as threatening its very existence, but is more accurately seen as opening its present to its past and to its future. (Antin, 1987, p. 479)

The noun *le genre* is an everyday sort of word in French, meaning "kind", "sort", "type" or "category". *Ce n'est pas mon genre* simply means "It's not my sort of thing". The allied English adjective 'generic' has some specifically mathematical overtones, while the cognate noun 'kind' has links to those ancient words 'kindred' and 'kin', evoking Wittgenstein's notion of 'family resemblance'.

What *kinds* of mathematics are there? And what are possible bases for distinction or grouping, what are some salient features that could be stressed or ignored? One way, important both to libraries and to *Mathematical Reviews*, is by means of the traditional yet still-evolving categories such as 'geometry', 'algebra', 'calculus', 'analysis' and 'number theory' – though these can generate turbulence at the boundaries, as well as increasingly requiring hybrids: algebraic geometry, topological algebra, analytic number theory, geometric topology, and so on.

Right from the time of Aristotle, there has been an emphasis by guardians on the need for purity of such kinds and strong injunctions offered against mixing them. And also right from those ancient times there has been a comparable tendency by others to do just that.

> Shakespeare satirized the rigid genre critics of his era in Polonius' catalogue (*Hamlet*, **II**, ii) of types of drama: "tragedy, comedy, history, pastoral, pastoral-comical, historical-pastoral, tragical-historical, tragical-comical-historical-pastoral, ...". (Abrams, 1988, p. 73; entry under *genre*)

What is it that distinguishes geometry from algebra, for instance? Is it the content, that is the 'objects' studied and their properties perhaps, or possibly the ways of thinking about them or arguing about their properties, or even how they are talked or written about? Might it be something about the form, e.g. the required presence of 'algebraic' symbolism or the necessary absence of 'geometric' diagrams? Is it perhaps to do with the function of the mathematical language (e.g. specific ways of expressing and manipulating generalisations) that makes something algebraic or maybe the role of geometric language in supporting a proof's imagery that renders something geometry? Could there even be a distinction made on the basis of hemispheric functioning or even more localised activity within specific regions within the human brain?

For a fascinating and detailed account of possible bases for a cogent distinction between geometry and algebra, see Tahta (1980). Tahta argues that geometry is unstable, that it keeps turning into algebra: “the geometry that can be told is not geometry” (p. 7). His piece makes explicit use of Gattegno’s (1965) claim that geometry involves an awareness of imagery while algebra draws on an awareness of mental dynamics independent of a particular content. Gattegno also observed in this piece that there is no geometry without some inherent algebra and noted “the extreme instability of the dynamics of imagery” (p. 22).

In a letter sent in 1980 to geometer Marion Walter, David Wheeler wrote:

> Another point, which I haven’t yet sorted out for myself, relates more particularly to geometry. There seems to be a sense in which geometry is more concrete, more special, and more relative to the individual, than algebra. This is both an asset and a liability – it’s an asset because when the situation is concrete enough, the criteria for knowing whether you’ve been successful or not are embodied in the situation (can you flip the shape over and it fits, say); they are not matters of convention as they are in written mathematics. But it’s also a liability in that when you haven’t got concrete objects at your disposal, you may not even know how to begin at all; there are no *rules* to apply, as there are in algebra. This last indicates the tremendous importance of imagery, which is where all non-concrete (non-tangible) geometry must begin.

Clearly, there is still much at stake in attempting to make such discriminations and there are various means of approaching the question.

A second quite different set of terms for dividing up (pure) mathematics comprises the following categories: definition, theorem, lemma, corollary, axiom, conjecture, proof, ... where each refers to the *function* played by an element in a developed, deductive theory. In *Proofs and Refutations*, Lakatos (1976) strikingly explored how these categories significantly interact in ways rendered invisible by traditional deductive, written accounts of mathematics, as can be seen in higher-level university textbooks and professional mathematics journal articles.

A third potential way of cutting up mathematics is to agree it is primarily written and then try to find bases (whether of form or function) for distinguishing and grouping types of writing into different kinds. The question subsequently arises as to whether any observable differences are purely superficial or are in some way necessary, produced in response to demands of the situation: does form always have to follow function? An initial list might include the textbook, the published journal article, the written expository lecture, the letter (or increasingly e-mail message), the popular account or the encyclopaedia entry, where each is also influenced by other non-mathematical examples of the ‘same’ form.

Finally, and with specific reference to Candia Morgan’s (1998) book *Writing Mathematically: the Discourse of Investigation* which constitutes our focus for the remainder of this essay, there is the question of how the functions and demands of (compulsory) mathematics schooling cut across each of these ways of partitioning mathematics. Should one wish to call these categories ‘genres’, are there what might be termed ‘school mathematical (sub-)genres’ which only exist inside classrooms, rather like Cuisenaire rods and Dienes blocks which, as fundamentally pedagogic objects, do not have an independent existence outside of school classrooms? [2] And, in particular, is the 'investigation report', the central focus of Morgan's textual analysis, such an instance?

In connection with the foregoing discussion, Morgan tellingly points out:

> nevertheless, the extent of the identification of mathematics with its symbol system is very likely to be significant in its effect on readers' interpretations of the texts – even to the extent that the presence or absence of symbolism may determine whether or not a student's text is considered to be 'mathematical' at all. (p. 14)

Pimm (1992) attempted to detail certain features of the classroom situation of *oral* 'reporting back' on mathematical investigations by students to the rest of the class, identifying certain features and constraints of the situation which led to teacher tensions (such as wanting the students to speak for themselves, yet wanting to shape what was said for educational ends). Far more extensively and systematically, Morgan has taken for her territory the coursework write-ups produced by secondary school students working on such tasks and subsequently sent off for external marking, as well as the declared and enacted beliefs of 'coursework moderators' (usually practicing teachers) employed to 'moderate' such student texts.

But before launching into a more detailed discussion of aspects of Morgan's book, we return to the related background issue of written genre.

On genre and writing

In artistic or literary arenas, genre has the linked additional sense of 'style' or 'form', with genres used as a basis for distinction and classification. Some literary distinctions embodied in genres (such as between lyric, epic or narrative, and dramatic poetry) are predicated on the presence or otherwise of a narrator and to what extent some or all of the text contains first-person or third-person characters speaking for themselves (see Abrams, 1988). [3]

There are some fearsome pedagogic questions in all this apparently scholastic nit-picking. In coming to know mathematics, are these categorical distinctions more conventional or fundamental in nature (a result, in Plato's striking image, of 'cleaving reality at the joints' perhaps?). If so, how will that affect how they might be profitably encountered? Do students need to experience this function or purpose *before* they are able to understand the conventional form? Or, by coming to terms with aspects of a form first, might they not then be able to gain an appreciation for why it is the way it is? Are genres and their divisions simply conventional and contingent? (See Hewitt, 1999, for discussion of a related question.)

How, then, is a relative novice, even when guided by a teacher, to come to grips with the bases for the distinctions? Are there surface features that might be explicitly alluded to associated with each? Might they be explicitly and systematically taught and, if so, would that be of benefit to some students who might otherwise not notice them? And, pragmatically, would their subsequent written work, in consequence, be assessed differently?

In the background to this discussion lie a set of debates about the teaching of language and literacy which have been raging for considerably more than a decade (the so-called 'genre wars' – see, for instance, Reid, 1987 or Cope and Kalantzis, 1993). Renewed attention to questions concerning language's interaction with written mathematics over the past decade has brought them into greater prominence in mathematics education.

On the one (polarising) hand, genres and explicit teaching of their features can be seen as a straightjacket, stifling originality and creativity, by substituting attention to the form for attention to the content. A mathematical instance could be seen by harking back to the nineteenth-century, teacher-insisted-upon need to use the exact lettering occurring in the text book when reproducing a Euclidean diagram in a geometric proof (although see Netz (1998) for reasons why this might not have simply been scholastic pedantry).

On the other (equally extreme) hand, lack of explicit attention to form and its crucial function in shaping successful communication leaves students to wallow in their own limitations, unable to partake of the cultural traditions into which they are (hopefully) being inculcated through their schooling. Morgan's book, to a considerable extent, documents this possibility for mathematical investigations, in part through the moderating teachers' (or anyone else's for that matter) disagreement over or unawareness of what defining features of the particular genre are.

Once again, we find ourselves with an instance of the grip Mason (1996), following Brousseau, terms *the* didactic tension:

> The *more* explicit I am about the behaviour I wish my pupils to display, the more likely it is that they will display that behaviour without recourse to the understanding which the behaviour is meant to indicate; that is, the more they will take the *form* for the substance.
>
> The *less* explicit I am about my aims and expectations about the behaviour I wish my pupils to display, the less likely they are to notice what is (or might be) going on, the less likely they are to see the point, to encounter what was intended, or to realise what it was all about. (p. 13)

Underlying the notion of *genre* lies the possibility of grouping members of a class into various named categories (whether specified on the basis of characteristic features of form or function, whether seen as fundamental or simply conventional). And a central pedagogic question with regard to individuals becoming more attuned to the discriminations inherent in such a categorisation is whether or not having their attention explicitly drawn to the specifying features is helpful.

Halliday (1978) writes:

> languages have different patterns of meaning – different 'semantic structures', in the terminology of linguistics. These are significant for the ways their speakers interact with one another; not in the sense that they determine the ways in which the members of the community *perceive* the world around them, but in the sense that they determine what the members of the community *attend to*. (p. 198; *italics in original*)

Genres also serve to highlight different patterns of meaning. Linguist Michael Halliday moved from the U.K. to Australia in the late 1960s and it is from there that much writing on issues of genre and schooling has subsequently emerged, specifically focused on examining ways of paying attention to genre features in formal education as a core activity in students becoming more widely and more successfully literate. Much of this work (e.g. Martin, 1989; Halliday and

Martin, 1993) is also rooted in questions of school systems developing greater equity by means of students gaining access to linguistic–cultural capital.

Since at least the beginning of the 1990s, the term *genre* has occasionally made an explicit appearance in mathematics education writing. Its presence reflects a growing return to the issue of aspects of form and the written in mathematics teaching and learning after more than a decade of attention primarily focused on issues of *spoken* mathematical language (code word: 'mathematical classroom discussion'). Marks and Mousley (1990), explicitly drawing on the genre work of Martin, give a broad range of examples of what they see as genres in their article, as well as summarising these into more general classes, also called genres:

> In solving problems, writing reports, explaining theorems and carrying out other mathematical tasks, we use a variety of genres, many of which are common to expressions of other language. Events are recounted (narrative genre), methods described (procedural genre), the nature of individual things and classes of things explicated (description and report genres), judgements outlined (explanatory genre), and arguments developed (expository genre). (p. 119)

And in a follow-up article, Solomon and O'Neill (1998) nicely draw out the difference between one genre at work within another (mathematical within epistolary, or within journal, with their different temporal and pronominal structural features), as opposed to it simply all being seen as mathematical 'narrative'. Their disagreement is with a genre meta-category called 'narrative' and claims by certain mathematics educators, e.g. Burton (1996), that mathematics writing in school *should* move heavily toward the narrative.

Solomon and O'Neill's work wonderfully exemplifies one of the opening injunctions of mathematician Norman Steenrod in his contribution to the American Mathematical Society pamphlet of essays entitled *How to Write Mathematics*.

> In this endeavor [trying to offer 'criteria for the excellence of an exposition'], I shall need to distinguish sharply two parts of a mathematical presentation: the *formal* or *logical* structure consisting of definitions, theorems, and proofs, and the complementary *informal* or *introductory* material consisting of motivations, analogies, examples, and metamathematical explanations. This division of the material should be conspicuously maintained in any mathematical presentation, because the nature of the subject requires above all else that the logical structure be clear. (Steenrod *et al*., 1973, p. 1)

What Solomon and O'Neill make clear about the instance they analyse of nineteenth-century mathematician William Rowan Hamilton and his various writings about the discovery and key features of quaternions is the extent to which a central way of maintaining Steenrod's distinction in mathematics is *syntactic* and therefore, we would add, may not be all that near the surface awareness of many writers.

Dixon (1987) speaks of Moffett (1968) distinguishing 'levels of abstraction' in the subtle verb tense shift from 'What's happening?' to 'What happened?' to 'What happens?'. The continuous present of Moffett's third level is part of the detemporalisation of mathematical statements, wherein they are caused to speak their timeless character. (See both Solomon and O'Neill, 1998 and Rowland, 1999, 2000, for more on this.) Time as a genre element in mathematics seems worthy of much further attention. This seems independent of whether it is marked by verb-tense or other features in different situations and languages (it is striking how many of the logical connectives mathematics relies on also have a temporal sense as well) or signalled by its forcible absence by the identifying of 'detemporalisation' (see, for instance, Balacheff, 1988) as a central process at work in the written expression of 'mathematics'.

On student mathematical investigation and the examinable texts that result

Nominalisation, another of the features Morgan focuses on as an element of written mathematical discourse, is related to detemporalisation. Activity, with its particular place in time, appears static when represented by a noun. The central tension identified in Morgan's important book is closely allied with the nominalisation of investigative activity into the notion of 'mathematical investigation'. In his chapter entitled 'Evaluating mathematical activity'. Love (1988) has documented well the snaking senses of this term 'investigation' over the past three decades, metamorphosing from an activity-prompting verb 'investigate such-and-such' to a stationary, examination-target noun 'an investigation'. Love refers in a concluding section to the problem posed in Yeats' most aptly-titled poem 'Among school children':

> O body swayed to music, O brightening glance,
> How can we know the dancer from the dance?

he identifies this problem as the signal flaw shared by a range of attempts to describe mathematical activity (some of which have been explicitly incorporated into assessment rubrics).

> In attempting to define the process we are always looking at products. Such products may be written, spoken, enacted: but they are always *after* the event. It is inevitable, therefore, that the categories we create to describe the process are static impositions – products – on the process. This can be seen in the reification of strategies, where such strategies appear to exist as things, although they do not necessarily exist at the level of consciousness of the individual problem solver. (p. 259)

He goes on to identify twin dangers of such descriptions occurring in this way:

> first, that the descriptions arising from the products appear to imply that particular processes must, or should have happened. These assumed processes then are used as a means to describe the activity. Mathematical activity then becomes so identified with the processes, that children will be seen as engaging in it only in so far as they seem to exhibit aspects of the descriptions. Secondly, and even more disastrously, teachers' actions are affected – so that they teach processes or strategies directly in the belief that they are then getting their pupils to act mathematically. (p. 259)

We shall see similar complex concerns and dangers surface in Morgan's insightful and thorough analysis of the formal assessing of mathematical investigation, where the significance of 'getting it wrong' is even more pronounced. Morgan's book appeared ten years after Love's warning was written, the same decade during which the U.K. National Curriculum for mathematics was imposed, and when a write-up of investigations became a core component of the GCSE mathematics national assessment at age sixteen. Morgan's book is partly empirically based on the text production of U.K. secondary school students in response to specific mathematical tasks set as part of their coursework component for this exam. But Morgan also interestingly draws on the observations and evaluations of a group of teachers who are paid to 'moderate' them independently for the examination boards. She does this in order to be able to explore some of the surface text features assessors attend to when 'seeing value' in this writing.

In Morgan's chosen title, 'Writing Mathematically', *writing* is a verb, not a noun. Thus, she draws attention to the activity behind mathematical texts. With this focus in mind, her subtitle, 'The Discourse of Investigation', points to the dialogue and text that form the context of both investigative activity and write-ups of this activity. Although her primary concern seems to be the activity of writing, she also analyses the features of the texts coming out of investigation settings, seeing the texts as instances of a genre.

The tension between context-specific student activity and the treatment of texts as decontextualised objects runs through Morgan's book. Her interest is often directed away from the activity of students writing toward the evaluation of the texts they produce. Because students write these texts *for* evaluation and because evaluators judge the students' texts outside of and away from the context of the students' activity, Morgan's shift of attention seems appropriate. Her empirical research focuses more on the activity of the teachers who evaluate student writing than on the investigative experience of the students themselves. The experience of students is considered in some of her chapters that review the literature and, in her conclusions, she reconsiders what she announces as her primary concern, namely students' experience of writing.

The opening, introductory chapter frames the work within Morgan's own teaching practice in relation to changing national examination forms which allowed a 'coursework' component (since 1995 worth 20% of the total mark), produced during the two years prior to the conventional, timed 'high-stakes' GCSE exam. She situates her work within two broad themes: secondary school mathematical language and assessment of students' mathematical activity. Her investigation of mathematical writing was born out of her sense that, as a classroom teacher, she was not adequately equipping her students to communicate their mathematical ideas successfully.

> I was unhappy with the quality of the written work that my students produced. [...] although I felt able to write competently myself, I did not have the knowledge about language or the skills in teaching writing that would enable me to help my students to communicate their mathematical activity more effectively in writing. (p. 1)

Her conclusions should come as no surprise: the problems she uncovered in her own teaching practice are shown to be widespread. She is disturbed by the apparent complacency within mathematics education discourse relating to student writing and finds problematic a lack of detailed knowledge about the various forms of writing.

Morgan sets the tone in this introductory chapter for a struggle against a mathematics education tradition that favours rigidity – a fight against a tradition to which she herself belongs. She is already worrying about misinterpretation of her key terms, 'appropriate' and 'effective', as she describes how she will make use of interviews with teachers:

> to draw some conclusions about what may be appropriate and effective mathematical writing in the particular context of reports of investigations. (p. 6)

In a footnote, she describes her intention not to "suggest that these have any absolute meaning. Rather, they must always be seen as relative to the particular context" (pp. 6-7).

Although Morgan does not discuss alternative interpretations of ‘appropriate’, her footnote prompts reflection on the different ways one can evaluate mathematical activity. *Appropriateness* describes activity that is fitting for its context: it can be contrasted with *rightness*, which describes activity that aligns with a norm. Rightness as an explicit goal has somewhat fallen out of favour in the West, perhaps because of a sense of the marginalisation that is often the result of humans judging others according to rigid standards.

In mathematics education, open-ended investigative activity mirrors the more general movement to avoid things over-rigidly defined. Even for mathematics educators who wish to avoid the apparent hegemony related to rightness, it is difficult to avoid exposure to it amidst *right* angles, *normal* lines, curriculum *standards*, *true* conjectures and *right* answers. The fixation on single right answers in mathematics classrooms and their absolute assertion – rather than the giving of reasons for them to be so – Alrø and Skovsmose (1996) term a form of “bureaucratic absolutism” (p. 5).

Judgements of appropriateness or fittingness need to be made in relation to the context. While considering the context’s traditions of suitable activity, individuals in the present apply these traditions to their unique contexts and come to individual decisions about what is fitting in the ‘now’. Although the concepts of rightness and appropriateness have different nuances, they are closely related – each implying the other to some extent. They both reflect an interest in the present, but the standard of effectiveness, which Morgan also mentions, is interested in the end result. Rightness and appropriateness have deontological concerns – the importance of duty outweighs concern for effectiveness. Effectiveness has teleological concerns – the ends justify (but also confirm) the means.

In her second chapter, Morgan reviews mathematics discourse literature. She begins by describing some characteristics of the mathematics register in general, but locates her interest in those features characteristic of ‘appropriate’ mathematical texts more than in features that mark text as specifically mathematical. She continues with her review of literature about mathematical texts by pointing out sources for her readers interested in the writing advice given within professional discourse, characterising this discourse as impersonal and based on deductive reasoning. After Morgan describes the professional discourse, she compares it with features of school textbooks. She surveys a wide range of the literature and exemplifies particular characteristics of mathematics discourse by means of a specific article published in the *Journal of the London Mathematical Society*. The two-page textbook account seemed relatively thin in comparison (though see Morgan, 1996). [4]

Morgan uses Richards’ (1991) distinctions between domains of discourse: the logic of discovery is present in research mathematics (done by professional mathematicians) and inquiry mathematics (carried out by mathematically literate adults), while in journal mathematics (articles written by professional mathematicians for their peers) and school mathematics any logic present is necessarily reconstructed. Her interest in the professional discourses of mathematicians is attributed to others’ claims that investigative activity is similar to the work of professional mathematicians. Yet, her analysis here focuses only on the reconstructed aspect of the professional discourse, not on the activity which lies behind the journal articles.

Morgan looks critically at the connection between student activity and writing in school, but does not look critically at the comparable connection in the professional discourses. Although professional mathematicians write impersonally, in the ‘timeless’ present tense, their articles are part of on-going conversations within the field, unlike texts written by school students for evaluation. For instance, Hanna (1989) contends that the form of the text in the professional discourse is relatively unimportant compared with its place in the on-going conversation:

> the acceptance of a theorem by practising mathematicians is a social process which is more a function of understanding and significance than of rigorous proof. (p. 21)

For educators interested in rightness, the connection between students writing and mathematicians writing may seem relatively important, though it does seem to presuppose that mathematics education *should* be guided by professional mathematicians' practice. Rightness tends to ignore context. With an interest in appropriateness, context is paramount. Thus, much more translation is necessary in the comparison between a mathematician writing and a student writing. The depth of connection between the two is dependent on the extent to which professional mathematics discourse is seen as *the* tradition behind written classroom mathematics, the one to which it *should* aspire and, reciprocally, which it is seen as approximating.

The book's third chapter reports on the writing-to-learn literature. Morgan notes that most of this research on the value of writing for learning mathematics focuses on student journal writing. One exception is reported: Mason *et al*. (1985) give some suggestions for improving thinking and writing in the context of mathematical investigations. Despite growing international interest in mathematical writing and communication, Morgan laments that there has been no significant change in the kinds of writing in which students are engaged. She is not convinced by the largely anecdotal evidence used to support claims for the value of writing to learn, and questions:

> the 'common-sense' assumption that the writing provides a transparent representation of the students' intentions and hence of their understanding. (p. 35)

She describes similar concerns in Chapter 4, in which she critically examines the literature that explores ways of teaching writing in mathematics. The complacency she finds within the U.K. regarding the commonly-acknowledged problem of students' poor writing disturbs her. Internationally, she reports a general lack of attention to the *form* of student writing below the university level. Taking issue with many claims that writing develops 'naturally', she argues against the given evidence noting a lack of attention paid to the forms and means the cited researchers used to judge the quality of writing. She suggests that, with more writing experiences, students merely tacitly learn the features valued by their audience, who is ultimately their teacher.

This chapter presents the rationale for Morgan's interest in forms of writing. She claims that knowledge of these forms is essential for research on student writing, for teachers involved in assessing student writing and for teachers preparing their students to write for assessment.

> It is [...] crucial to identify those forms which may be considered 'appropriate' within a given genre of mathematical writing. This is the task that I attempt to undertake in later chapters of this book for the particular genre of reports of investigations. (p. 49)

Here she locates the context she has in mind for the word *appropriate* used to describe mathematical writing – it is in reference to the features of the genre.

This prompts us to consider other contexts for evaluating appropriateness. In the context of activity, appropriateness might be seen differently from within the context of a genre. Although genre gatekeepers, such as journal editors, might make judgements about the appropriateness of a text for distribution within their *particular* genres, their judgements do not suggest that the text is *generally* inappropriate. The activity of writing is different from the text it produces. Activity, even the activity of writing, can only be regarded as appropriate or inappropriate in its immediate, particular context.

This leads on to questioning the appropriateness of external assessment of open-ended mathematical activity. Coursework moderators who evaluate students' investigative write-ups, by virtue of their separation from the context of the activity, are not equipped to use the texts as a means of judging the quality of the students' mathematical activity. If their function is to consider the text apart from the activity, can they ask whether the text is serving its intended purpose? Yet, as Morgan notes, both teachers and examiners have always wanted to be able to infer, unproblematically, quality of thought from quality of writing.

In Chapter 5, Morgan locates the discourse of investigation by looking at three forms of literature related to it: the *official* discourse (e.g. curriculum documents), the *practical* discourse (advice guides for using and doing investigations) and the *professional* discourse (educators writing about investigations). These three discourses agree that investigations involve students creatively doing real mathematics set in the context of open-ended situations. They also agree on the importance of this activity's assessment, which can compromise both openness and creativity.

The intent of this chapter seems to be to explain the investigation tradition for those readers who may be unfamiliar with it. She notices that such familiarity is essential:

> in order to be able to interpret the significance of what students write with reference to the context in which their texts are produced and read. (p. 74)

Morgan's model reader is evidently someone familiar with the U.K. investigation tradition, but this chapter gives some help to the unfamiliar reader, actually serving, in part, to bring about her desired model reader.

In Chapter 6, Morgan provides a toolbox for the analysis of mathematical texts. She organizes her analysis of the literature that supports each tool according to Halliday's (1973) three meta-functions of language: the *ideational* (expression of the categories of one's experience of the world), the *interpersonal* (structuring relationships between the author and others) and the *textual* (structuring the interconnectedness of the text). She warns that any text should be interpreted in its complete form, because apparently contradictory features may exist.

In this review of the tools of textual analysis, Morgan reveals her awareness of the tensions inherent in her study, as she recognizes the impossibility of her aim to find generally appropriate forms of text for the investigation write-up:

> the individual's positioning within a particular social structure and consequent understanding of the nature of the genre within which she is writing makes it 'natural' for her to make [...] choices because they appear 'appropriate' to the task she is undertaking. They may or may not appear similarly appropriate to a reader, depending on the discourse within which that reader is positioned. The analyst, however, must stand apart

> from making such judgements as the concept of 'appropriateness' is itself socially constructed and is indeed one of the ideological concepts that is to be 'demystified' by the analysis. (pp. 97-98)

In Chapter 7, Morgan provides her readers with the opportunity to see particular text samples from classroom investigation reports. She locates two investigations in their curricular context and describes the official guidelines for external evaluation of student work on these investigations. Morgan uses the toolbox described in the previous chapter to analyse excerpts of student writing in response to these tasks. Because she breaks her own rule about the necessity of analysing texts in their complete form, she seems primarily interested in providing illustrative examples of the use of these linguistic tools.

In her eighth chapter, Morgan reviews the literature relating to assessment of student work that resembles investigative work. She seems disturbed by the literature's apparent interest in mark reliability, because moves toward reliability privilege less-creative work. She reiterates here her interest in finding appropriate forms for the genre but avoids a word she used earlier – *effective*.

> This notion of 'appropriateness' in relation to the assessment of investigative work needs to be explored. In particular, what is the nature of 'appropriate' forms of mathematical communication? (p. 130)

With attention directed toward assessment, effectiveness no longer seems to be an appropriate goal. Presumably, the 'effect' sought after by students aspiring to write 'effective' coursework scripts is a high mark. Writing appropriately for their context may seem to be unimportant to students interested in good grades. To achieve a high mark, students will want to know what kinds of writing their assessors will value (which of course, gives rise to another instance of the didactic tension).

In Chapter 9, Morgan details the design of her investigation of teachers' readings of investigation write-ups and shares some of the resulting data. She looks at different evaluators' assessment of particular write-ups and notes disparity. Assessment is dependable in the case of texts describing relatively routine mathematical approaches, but it varies significantly for unexpected or unanticipated write-ups. Students writing these texts cannot know their empirical audience.

> It is unclear whether the grade achieved by an individual student may be taken to be a measure of some general impression of his or her 'ability' or an indication of the extent to which the text produced matches the teacher's image of an ideal response to the task. (p. 148)

In Chapter 10, she uses her interviews with coursework moderators to outline the major empirical features that teachers look for during their reading of student texts and to construct a picture of the gross form of an 'ideal' coursework text. She reports that teachers favour tables as a sign of systematic work, abstracted diagrams (but not too many), algebraic generalisations (preferably with single-letter variables), a restatement of the problem, a narrative of the process (using verbs that describe thinking rather than action), explanation that implies causality and, finally, no misuse of the mathematics register.

What values underlie this set of preferences? An interest in abstraction can be seen in the preference for algebraic generalisation and in the kinds of diagrams preferred. However, as demonstrated by the preference for narrative, the abstraction needs to be grounded in the context of the students' particular process. Supporting abstraction in another way, the preference for tables seems to indicate the wish that students apply generalised problem-approaching procedures to their particular task. Unfortunately, only one such heuristic seems to be supported (see later for more on this point).

In Chapter 11, Morgan directs her attention to the assessment of write-ups that proved different from the expected. She explains how a teacher's own experience of a problem influences the assessment of student work. The evaluator tends to struggle when interpreting write-ups of unique approaches. Morgan suggests that evaluators construct an "explanatory narrative" to make sense of write-ups that are new to them. The explanation of more routine approaches is bolstered by the teacher's similar experience. Because the shape of the evaluator's previous experience is a significant factor in the assessment of the quality of the work, external assessment cannot be 'fair' for open-ended work. She also mentions a related tension – while assessment looks with disfavour upon mistakes, the discourse of investigation values mistakes and tentative conjectures as potent beginnings of new exploratory paths.

From our experiences with students undertaking mathematical investigations in Canada, a practice which is far from widespread nor institutionalised there (see, for instance, Wagner, 2002), a student's sense of what is intuitive and what is not seems to be particular to his or her experience with the investigation. Even a unique approach can seem intuitive to the one who employs it and thus the written explanation might seem spare or even deficient to someone for whom the approach is new. Routine approaches (i.e. similar to ones the assessor has considered) might well seem equally intuitive to both writer and reader and each party is likely to come to the same conclusions about what is important or 'necessary' to write.

These problems again suggest the inappropriateness of external assessment for open-ended tasks. Edgerton (1996), like Morgan, is disturbed by the assumption of the transparency of language, and writes about the violence of evaluation, although outside the discourse of mathematics education.

> If we learn to believe that our lenses are universal and fixed, we can not translate, hence we cannot connect with one another nonviolently. (p. 35)

Her alternative to the judgements that are inherent in evaluation is love, described in this way:

> listening is love; love pays attention. Thus it occurs only in an open system – that is, an interactive, intertextual system. (p. 69)

It seems that Morgan herself is trying to bring this kind of respect for students' mathematical activity into the discourse of investigations, which is also an open system. In the context of externally-validated assessment, to what degree is Morgan's hope realistic? And, as Morgan seems to be asking, how can teachers fully attend to their students in a mathematics classroom for which external assessment is mandated?

In her final chapter, Morgan summarizes the contents of the book and concludes that there is need for what she calls a "critically aware mathematical writing curriculum".

> Knowledge about the different effects that various linguistic choices can achieve can provide students with the power to manipulate their own use of language to produce such effects deliberately, for example, to name their variables consistently or to use the present tense in order to be seen to be 'more able'. (p. 209)

She does not go into detail about the shape she envisages for such a critically-aware curriculum, but seems to suggest that teachers explicitly tell students which forms of writing are valued so that students can incorporate these forms into their writing. This approach seems antithetical to the values supporting investigation. She seems caught in the difficult predicament common to people who care deeply for others and are looking for ways to do justice in a system that seems to be more concerned with reliability than with justice.

By asserting the need for critical language awareness without discussing its specific nature, Morgan leaves us wondering what kinds of classroom tasks could direct students to *explore* the effects of various forms of writing. They might involve historical mathematical texts, current text from the professional mathematics discourse, aspects of students' own textbooks or samples of other students' write-ups. The students' exploration would necessitate consideration of the purpose for and audience of mathematical texts and might lead to different conclusions about what forms are most valuable. However, if their own writing is modelled on their thoughtful consideration of other people's mathematics texts, alas they may adopt forms that are not valued by their external assessors.

Morgan's own book as a text

Without question, this book represents a significant contribution to mathematics education in the area of language and discourse studies. Morgan has an immense command of a wide range of related literature, much of which is well brought to bear on her project. Anyone working in this area would do well to consult Morgan's text and bibliography. That said, this work seemed somewhat reference-heavy in places, reading more like a thesis than a book. Like a considerable number of the books in the Falmer *Studies in Mathematics Education* series, this book does indeed derive from Morgan's doctoral work. [5] In terms of the thesis genre, there is an expected move from general to particular and back out to general once again. What do we find here?

As we have seen, the book comprises a dozen chapters the first five of which move us fairly smoothly in from the general to the more specific in terms of discussing characteristics of written mathematical texts, writing in the mathematics classroom and questions of learning to write mathematically at the outset before narrowing in Chapter 5 to the public discourses ('official', 'practical' and 'professional') surrounding the school mathematical phenomenon of 'investigation'.

Chapter 6, however, seems somewhat sequentially misplaced, at least in terms of the expectations generated by the genre, to the extent that it applies a general set of techniques from critical linguistics to mathematical texts of all sorts. Then Chapter 7 details the specific texts themselves which at first glance would seem to be likely focal candidates for Morgan's personal analysis (and Paul Ernest in his series introduction claims she "investigates both secondary school children's writing and also how it is 'read' by teachers" (p. x), which is only somewhat the case). For it is here Morgan takes an interesting and significant turn away from the texts themselves (and hence their genesis as records of student experience) toward their function in the national assessment system.

Chapter 8 is more general once again, on teacher assessment, before focusing in on her study of a specific group of teachers assessing these particular texts in Chapter 9 and 10, prior to the expected return to greater generality in the final two chapters on assessing difference and moving towards a critically aware mathematical writing curriculum.

The term ‘genre’ shows up first on p. 8 where Morgan describes texts written by students as reports of their mathematical investigations as being instances of one. As with many other writers, the fact that a claim is being made here is unacknowledged. (One notable exception to this observation is Gerofsky, 1996, 1999, 2002, who is at great pains to justify her claim that mathematical word problems do indeed comprise a specific genre.)

The genre issue returns in Morgan’s discussion of the work of Marks and Mousley (mentioned earlier), where she claims that “in order to develop mathematical literacy, children need to learn a wide range of the types of writing used in mathematics” (p. 39). And yet the next paragraph details work on the ‘genre’ of two-column proof. It is as if everything is a genre and there is no sense of requiring comparability of scale or scope. We see this as one of the areas in need of much work, namely stratifying what may reasonably be called a ‘genre’ in mathematics education.

One considerable attempt to explore a variety of specified genres (with classes of nine- and ten-year-old children), as well as places where other non-mathematical genres intertwine in the social context of the writing (which jointly result in what is termed ‘paramathematical’ writing), can be found in Phillips (2002). She explores the strong interrelationships among form, purpose, content, audience and voice. finding considerable reciprocal influence and even mutual constitution. One of the sites she extensively explores is that of her own students’ writing of a mathematical textbook, standing on its head the following observation of Morgan’s:

> Nevertheless, since the text book is the dominant model of mathematical writing available to school students, it is of interest to consider the extent to which students adopt text book language in their own writing. (p. 19)

In particular, Phillips reports ways in which many formal elements of the textbook genre stayed more constant than either the language or the tone used and has some fine examples of particular student voice and paramathematical writing being deployed for pedagogic ends.

In conclusion, any piece of writing that is about writing style is at particular risk of having its own style attended to. For instance, in the introduction to *The Place of Genre in Learning: Current Debates*, Reid (1987) draws explicit attention to “the genre(s) of writing-about-genres” (p. 3). In the opening chapter, Morgan writes:

> A formal, impersonal style, including an absence of reference to human activity, is one aspect that mathematical writing appears to share with many other academic areas, in particular with writing in the sciences. In this section, the nature of academic mathematics texts is considered, taking into account the views of mathematicians themselves and research concerned with scientific academic texts in general. Examples from one academic mathematics text (Dye, 1991) will be used to illustrate the ways in which some of the characteristics are manifested in a mathematical context. (pp. 11-12)

The latter two sentences actually themselves exemplify the claim present in the first and reflect the genre of doctoral dissertation (as do phrases like "As was seen in Chapter 3"), and illustrate ways in which linguistic forms can be used to suggest features of the content of which the language itself speaks.

A return to the *avant garde*?

We started this review with a quotation from David Antin, where he specifically makes mention of the role of the *avant garde*. Within post-W. W. II mathematics education in the U.K., the use of mathematical prompts beginning 'investigate such-and-such' constituted an important instance of *avant-garde* mathematics teaching, as regularly reported in the pages of the Association of Teachers of Mathematics journal *Mathematics Teaching*.

In a more recent instance of such writing, Hewitt (1992) identifies examples of aberrations, where the institutionalisation of 'investigations' has perverted the original pedagogic intent behind offering students such tasks in school. In particular, he points to an instance of the mathematical education problems that can arise when teachers over-value certain forms *qua* forms. As Morgan's work illustrates, these forms can be taken as signifying mathematical activity or standing as an emblem of mathematics itself. In Hewitt's piece, the privileged form is the drawing up of numerical tables, independent of their particular value in relation to the specific purpose and context of the task at hand. (Polya's (1945) heuristic advice of 'draw a diagram', at once both too general and too specific, can be viewed in a not-dissimilar light.)

One of the challenges of any *avant-garde* movement is to survive while not being completely absorbed into the mainstream, for fear of both perversion of intent and neutralisation of the possibility for innovation or change. As Love's (1988) piece documents, the Cockcroft Report (DES, 1982), arising from a U.K. government commission into the teaching of mathematics at school level, played a significant role in 'normalising' such work – and the history documented in Morgan's book of 'investigations' being co-opted to constitute a significant assessment task continues the tale. Morgan's own work provides an insightful look at pedagogic practices in relation to examiners' perceptions of what constitutes worthwhile mathematical writing, while also offering her own sophisticated look at such applied pragmatic–linguistic concerns.

Streams and reservoirs, genetic or otherwise, fill and empty over time, becoming renewed or dying out: the sources and tributary outlets change or become absorbed. Their history is continually emerging just as their history is continually forgotten. The constantly recreating and regenerating family that is mathematics teaching – together with its goals, practices and purportedly-linked rationales – is open to its future as well as needing to be reminded of its past. The once-*avant-garde* activity of mathematical investigation has become or is in the process of becoming an active element of the mainstream present, both in the U.K. and elsewhere. Its on-going development is one to watch and participate in with interest and Morgan's book provides a careful and thoughtful companion to accompany us on this journey.

Notes

[1] Morgan, C. (1998) *Writing Mathematically: the Discourse of Investigation*, London, Falmer Press, 232pp. (ISBN: 0-7507-0811-5 hbk; 0-7507-0810-7 pbk)

[2] An even more important question has to do with categorisation as an activity and the extent to which the distinctions are based on purpose (similar to Lakatos' account of the centrality of definitions in relation to theorems) and the stability of forms arising from a continuity of purpose (much as with a comparable set of features specifying the form of mathematical word problems - see Gerofsky, 2002).

[3] One of the points Abrams (1988) makes about Roland Barthes' and others' structural criticism produced in the 1960s is how a genre can be:

> conceived as a set of constitutive conventions and codes, altering from age to age, but shared by a kind of implicit contract between writer and reader. These sets of conventions are what make possible the writing of a particular work of literature, though the writer may play against, as well as with, the prevailing generic conventions. For the reader, such conventions function as a set of expectations, which may be controverted rather than satisfied, but enable the reader to make the work intelligible – that is to *naturalize* it, by relating it to the world as this is defined and ordered by the prevailing culture. (pp. 73-74)

[4] There is also an extensive account of features of one U.K. school mathematics series in an adjacent volume *The Sociology of Mathematics Education* in the Falmer series written by Paul Dowling (1998).

[5] Of the first nine books in the series, five are based on U.K. doctoral dissertations in mathematics education.

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