Subject to mathematics

David Wagner

Pre-print (text submitted after peer review, before formatting etc.)

Wagner, D. (2022). Subject to mathematics. *For the Learning of Mathematics, 42*(1), 35-39. Available at https://flm-journal.org

In a mathematics classroom, who is subject to whom? Are the students subject to mathematics, governed by the rule of mathematics? Or, is mathematics the subject of the students, a system that is examined by students. In other words, is a student the subject of mathematics or is mathematics the subject of students?

In English, the word *subject* is slippery because its meaning is often unclear. The subjects that students study or practice in school are things like mathematics, language, or science. The word *subject* also comes up in relationships of authority: people under a monarch's power are called subjects. In this way, students can be the subjects of mathematics or their mathematics teachers.

In this paper I meditate on authority and subjectivity in mathematics education. Drawing on the practical rationality (Herbst & Chazan, 2003) of mundane and explicitly political contexts, I argue for mathematics education practices that help us understand how people use mathematics to shape our world. It is important for citizens to understand how we are subjugated by mathematical systems—systems designed and sustained by people. The first context comes from my experiences teaching school mathematics in the 1990s, when I introduced random processes into my everyday classroom practices. The second example is the one that prompted me to write this reflection. I see a need to explain mathematics to adults—specifically, available options for voting systems. Choices for political action are limited by the general public's understanding of mathematics in action, so I look to school mathematics to equip citizens adequately.

Subjectivity

Within mathematics education the word *subjectivity* usually comes up as an opposite of *objectivity* to refer to situations in which a person's perspective is significant—where a person exercises agency in interpreting a situation. An example of this applies to the stories I share below; they are subjective accounts of my experiences that rely on my memory and interpretation. There can be no objective account of these situations. I tell the stories here to provoke reflection, not to be accurate. But I try to be truthful. Your ability to visualize situations like the ones I describe is more important than the accuracy of my accounts. By contrast, in mathematics we usually consider objectivity to be a virtue (perhaps the primary virtue). Mathematical generalizations should be true no matter whose point of view one takes.

Critical theorists sometimes use the slippery word *subject* (and its derivatives) to play with conceptions of self, especially in contexts of power. For example, Gramsci's notion of subalternity relates to the way subjectivity works. "Structure ceases to be an external force which crushes [people], assimilates [them to themselves and makes them] passive; and is transformed into an instrument of freedom, an instrument to create a new ethico-political form and a source of new initiatives" (Gramsci, 1971, p. 367). As I understand this notion of subjectivity, Gramsci recognizes that the forces people use to resist a structure were nevertheless acquired and enabled by their experiences within that and other structures. In this paper, I argue that mathematics education should make this process described by Gramsci more explicit. Instead of studying the power of mathematics from the outside, understanding is built on feeling mathematics from the inside.

Some mathematics educators in traditions of critical theory have challenged simplistic conceptions of power and authority. Students can at the same time be subjects of mathematics and have mathematics be subject to them. Brown (2008) described how "Lacan's conception of subjectivity, whilst complex, does provide a way of thinking differently in which 'teachers', 'students', 'mathematics' and the frameworks that define them (curriculums, policy initiatives, research frames, learning theories, public expectations, employer demands) are conceptualised as mutually evolving entities resulting from the play of discursive activity" (p. 243). Similarly, but drawing on different theorists (Deleuze, Guattari, Latour, and others), de Freitas and Sinclair (2012) noted that "Post-humanist theories of subjectivity […] have shown how subjects are constituted as assemblages of dispersed social networks, and have argued that the human body itself must be conceived in terms of malleable borders and distributed networks" (p. 136).

The substantial and growing body of mathematics education scholarship that discusses authority and agency tends to valorize practices that position students as active decision makers, and thus promote practices that position students as active subjects. I have usually taken this stance too in my writing about authority and agency. However, in this meditation on subjectivity, I take an apparently opposite approach. I want to consider the value of practices that position students as subject to mathematics. My hope is that such practices can help students understand the way mathematics works. More particularly, I want them to see the way mathematics is worked by people in power. My hope is to equip possible resistance to powers and to enable possibilities for alternative power structures.

Experiencing randomness

When I was a new school mathematics teacher in the early 1990s, I participated in a potluck dinner with friends. This is where everyone brings food to share. Often potluck dinners are manipulated to ensure a balanced meal, with all the courses represented—appetizers, main courses, and desserts, for example. For this potluck we had decided in advance on a "true potluck"—no manipulation. When we arrived, we laughed upon seeing that everyone had brought dessert. We decided we were young and healthy so we could eat this one decadent meal without consequences.

While we feasted on sugar in its various forms, the background music was provided by the host's new state-of-the-art CD-changer on shuffle play, playing random songs from five CDs. Someone noticed that it had played three consecutive songs from the same CD and asked whether this meant something was wrong with the CD player's random function. A giddy argument followed around the question of whether we should be surprised by a random algorithm playing three consecutive songs from the same CD. We counted the songs on each CD. We performed mental calculations as we feasted. We justified our calculations. We were compelled to do this by the context.

Incidentally, the host told us later that he had phoned the CD manufacturer to tell them about our argument. He found that the randomization algorithms were recently changed to make consecutive songs from the same CD less likely because too many customers were complaining about malfunctioning randomness! I noted that their manipulated randomization algorithms could further undermine the public's understanding of randomness.

This dinner with random food and random music provoked me to pay attention to the way people experience randomness. I know now that there is rich literature on conceptions of risk and probability as far back as Fischbein's (1975) work. In the 1990s I would have been especially interested in the research on misconceptions of probability because I noted that even my very educated friends carried misconceptions about randomness. This helped me understand why my students and my math teacher colleagues struggled with probability. Randomness and probability go hand-in-hand; people cannot understand one without the other. I think the most significant reason for the importance of understanding randomness these days comes in the interpretation of reports on and experiences of weather, climate, and climate change.

After that potluck dinner, I decided that a solution to the problem of misperceptions of randomness was to include experiences of randomness in my math classroom. I reasoned that my students would understand random events better if they were subject to randomness. Before this, my students and other students had no stake in the math questions that were typical in mathematics textbooks. For example, why would they care about the colour of marbles pulled out of a bag?

Random homework checks

I introduced random homework checks into my math class routines. Some of my colleagues (including me before this) checked every student's homework every day. My new practice was to check only a sampling of homework. I numbered six regions in my classroom and rolled a die at the front of the class each day to identify in which region I would check students' homework.

Of course (or, I might say, "a working knowledge of probability makes us aware that") sometimes the same students would have their homework checked three days in a row. Or more. Students would often complain. Unsurprisingly, they or I would ask what is the probability of this happening. This question did not feel theoretical. It was practical.

Students wanted to know. They needed to know. And they argued with each other about the probability.

These arguments featured good mathematics well before the curriculum expected the students to investigate the questions they were discussing. Yes, middle school students can handle this mathematics. Misconceptions surfaced, but other students challenged the misconceptions. For example, someone might feel “safe” from a homework check because they were selected yesterday, or because they had been selected on two consecutive days, but their friends would remind them that they were no safer than they would be any other day. This was verifiable empirically. Or, some students would complain that they’d been selected disproportionately often. This claim could be tested as well but it is mathematically tricky to test.

I played with the student complaints. I would ask to whom they should direct their complaints. Sometimes I let the complaining students roll the die next time: to their frustration, they would sometimes roll their own region. (This happened one sixth of the time. Over a year this meant ‘often.’) Students argued about whom to blame. The one rolling the die? No. They philosophized about blaming *the system*. It was not very satisfying to blame the system, so they would conclude that it is most appropriate to blame the person who constructed the system. Me. Then I would offer that we switch back to the norm—full homework checks. No, they preferred the sampling. So now they saw their complicity in the system. This made blame difficult. The frustrations of the random results resurfaced almost every time a ‘strange’ occurrence happened. The students were learning that they should expect ‘strange’ occurrences.

Why did the homework sampling matter to students? The results of this random sampling would determine a student’s homework grade (which was a miniscule amount), but I invited students to ask me to check their whole workbook if they thought that the sampling was not representative of their homework practices. Significantly, through years of this policy on sampled homework no student ever asked for a full review of their homework practices in place of the sampling. When looking at their homework marks that were based on the sampling, some students told me that they suspected the full review might give them a slightly better mark, but they preferred to keep the sampled mark. They noted that any difference would not be significant to their overall mark. However, they noticed that the power of the homework mark was in the relationships. They cared what I thought about their homework practices and they preferred the ambiguity of a sample. They thought a full representation would make them look worse.

When the mathematical arguments about probability came up, sometimes I let the students argue amongst themselves regarding the mathematics and the impact of the mathematical systems. Sometimes I asked questions to help them consider cases that would expose misconceptions. I avoided telling them definitive answers to their questions about the probability, worried that my answers would kill the dialogue. Like my sugar-drunk friends at the potluck, my students seemed to take pleasure in provoking each other while arguing about the probabilities. They also took pleasure in satirizing the situation and in using these opportunities to fortify their positions in social relationships, like the students reported on by Dan Chazan in Chazan and Ball (1999). My students used mathematics for a range of their purposes.

Random seating

The other random practice I started following the potluck dinner was to introduce random seating in my math classrooms. I worried about relinquishing control over the positioning of my students but decided that the practical experiences of randomness outweighed these worries. All my colleagues designed seating plans for their classes, and so had I up to this point. We often had conversations in the staff room about which students work well together, which students should be kept apart, and which students benefited from being in the front or the back of the room.

With my new system, at the beginning of a new month each class created its new seating plan. A student would draw a map of the desks on the blackboard, writing a number on each seat. I had a coffee tin with slips of paper with these numbers on them. I held the tin above eye level and students came in turn to draw a number from the tin. The student at the board would replace the number on the map with the name of the student who'd drawn it. After everyone had made their selection the students moved to their new seats. As with the dice rolling for homework checks, I had decided that the randomness needed to be visible; the physical experience of participating in the randomization practices and the transparency of the methods motivated dialogue.

As with the homework checks, 'strange' eventualities would provoke (sometimes playful) outrage and argument among the students. For example, someone would be in the same seat as the previous month. "Hmmm. What are the odds of that?!", I'd ask. The conversation would sometimes go like this:

- There are 30 students in the class. What is the probability of **you** choosing the same seat (looking pointedly at the student who didn't move this month)?
- 1/30.
- What is the probability of **one of you two** choosing the same seat (pointing out two students)?
- 2 times 1/30.
- What is the probability of **someone** in the class choosing the same seat, anyone in the class?
- 30 times 1/30.

And then students would erupt with argument. The answer cannot be 30 times 1/30: 30/30 would mean certainty. But it is certainly possible for all the students to be in different seats from last month.

As with the homework checks, questions of blame arose. The students generally liked to have someone to blame for their misfortunes, but they preferred this system that made blame impossible or at least ridiculous. They did not wish for the alternatives—assigned seating or self-selected seating. They liked random seating: at month end, students would excitedly remind me that we get to change seats the next day. But as I look back, I try to remember which students showed this excitement. I had assumed that everyone liked it because some or many students said so. In hindsight, these experiments did not convince me that random seating is always best for the students and the health of the classroom interactions. I didn't always use random seating after starting the experiments. This makes it clear that I had criteria, I just don't remember what criteria I used at first.

Looking back, I recall students from marginalized groups arguing against self-selected seating. They wanted protection from the abuse of their classmates. I do not know how much actual abuse they experienced in my classroom, as I tried to be vigilant. But I knew the threat of abuse was real. It is easy for a straight, white, Canadian male like myself, to feel welcome and comfortable in any random location, but it is dangerous for us to assume that anyone feels this kind of safety. And it is dangerous for a teacher to impose a system that challenges the safety of students. This reflection relates to my most startling observation in this experiment: when students who should not have been sitting next to each other were randomly placed next to each other, they rarely exhibited the behaviours I was worried about. As I talked about this with some students, we speculated that the expected troublemakers[1] seemed to want to honour our trust in them.

Over the years when I have told other educators about this experiment, my observation of the positive social impact of my random seating practices is the one that has sparked their interest most. However, my reflections point to the importance of the reasons for imposing randomness and the significance of how the teacher mediates the randomness through conversations with students. I continue to experiment with seating and group composition, not to justify a claim about the superiority of random positioning, but to be thoughtful. In my university teaching currently I institute some random grouping, some assigned grouping, and some self-selected grouping. I want my students to experience the dynamics of the emergent social relationships and of randomness, but I am intentional (not random) about why a certain structure is most appropriate in a certain context. I worry about claims about the effectiveness of any system that places students in positions without consideration of their vulnerabilities and I acknowledge that I was relatively blind to this concern back in the 1990s.

My two experiments with introducing randomness into my mathematics classroom, along with the dialogue these experiments provoked, taught my students about distinctions between what the curriculum called 'independent events' (the dice rolling) and 'mutually exclusive events' (the seat selection: once a seat is selected it is no longer available), the impact of structural choices (especially mathematically-based structures) on social dynamics, the opportunity for designers of structures to use mathematics to hide from responsibility, and the way mathematics can be used to argue about such structures.

---

[1] When I was a high school student, I was a high performer but I too was sometimes seen as a 'troublemaker' who was actively separated from my friends to avert 'distractions' in the classroom. A dynamic like this is nicely described by Houssart (2001): troublemakers often carry truth and appropriately challenge the powers that be.

Structuring Representation

Currently, I am active in a minority political party. The governance committee of its Council is working on our system for electing Council members. We are considering a range of alternatives with the explicit goal of a “more representative Council”—leadership that reflects the diversity of the people. Our dialogue and my reflections on it point to the need for citizens to understand the mathematics of elections. I will make the case for math classrooms to do this by making students subject to forms of representation, similarly with my experiments subjugating students to randomness.

In Canadian politics (and elsewhere) first-past-the-post elections are the norm. The one who receives the most votes wins. A regional example exposes a problem with this approach: the Progressive Conservative party won a majority of the seats in my province’s Legislature in 2020 and thus the party now controls all legislation. However, they secured only 39.3% of the total votes cast. One can argue that the majority voted against them. Because of this problem, minority parties often favour ranked ballots, which offer a corrective to first-past-the-post systems. With ranked ballots, a person or party needs to have a majority of the votes to win.

In the context of a Council election, a first-past-the-post election for President means that the candidate with the most votes wins. Imagine two candidates A and B who represent the values of most members, and candidate Z with a different vision. In situations like this, Z often wins because the majority of members split their votes between A and B. With ranked balloting, members rank all the candidates including an option of ‘none of the above’. If no one has a majority of first-place rankings, then the name with the least first-place rankings is crossed out on all the votes, and there is a new count. This is repeated until someone has more than half the votes. This system averts the scenario described above where Z wins because the majority split their votes. Although ranked ballots are quite straightforward, the mathematics becomes a challenge when multiple people are being elected.

The Progressive Conservative majority described above is also influenced by another structure—the system of district boundaries. In many contexts, the people in charge of drawing political boundaries are guided by their partisan preferences. The process of drawing boundaries for advantage is called gerrymandering. Electoral districts can advantage a party without intentional gerrymandering. Mathematicians have developed approaches to judging the fairness of boundaries and to constructing fair boundaries (e.g., Klarreich, 2017). Choices to use these approaches (or not) are political.

In my party’s governance committee, some people want to ensure regional representation on the Council and thus have members elected from each region. This would give more weight to the votes from members in regions with fewer members, which is preferred by some and not by others. In any case, we would like to grow the party to develop a strong membership base in all the regions which would result in each member’s vote being relatively equal. Furthermore, the choice to highlight regional representation favours one kind of distinction over others. We also care about things like gender representation, linguistic representation, rural representation and representation of visible minorities. Thus some members of the governance committee have argued for ranked ballots to be counted in a way that honours the various diversities of the voters.

Mathematicians have developed systems for the election of a slate of representatives with fair representation. The usually-favoured approaches are variants of the Single Transferable Vote (STV), such as Scottish STV or Meek STV (named after mathematician Brian Meek who devised it). With STV systems, voters rank all the candidates. For example, when a membership elects members-at-large for their Council, there are usually multiple positions. Members rank all the candidates. The number of votes needed to be elected is found by dividing the number of voters by the number of positions and adding one to the result. Imagine 100 voters and five positions. A candidate is elected with 21 votes: $100 \div 5 + 1$. Candidate A may get 35 first place votes. This person is elected with 21 votes, leaving 14 surplus votes. These 14 surplus votes are then distributed by counting the second-ranked candidates in the ballots that had ranked A first. The different STV approaches have different systems for distributing the surplus votes.

With first-past-the-post elections a dominant majority chooses all five Council members. A fairer system would elect five regional representatives with ranked balloting, but a problem remains in the division of regions. How are the regions divided and why is regional diversity favoured over other diversities? With STV approaches to a slate of Council members, the distinctions are not predetermined, and Council leaders are elected with sufficient support (equal to the proportion of the membership divided by the positions).

Mathematics alone does not solve the problems. The voting body needs to understand and accept a system that can feel strange. I notice that people readily accept and acquiesce to prevailing systems and are suspicious of alternatives, saying they are difficult to understand. Thus mathematics education is important.

Analysis of different electoral systems could be interesting in its own right. Borba and Skovsmose (1997) have promoted this topic for mathematics classrooms. However, as noted in my Party's Council discussions, the consideration of electoral systems is interesting for nerds (yes, some of us self-identify as nerds) and it *becomes* interesting to others as they begin to understand the implications for the community they hold dear. Exploring systems can feel like a game or an academic exercise until we understand the stakes. Then it becomes a necessity.

It is interesting to note that prominent mathematics education conference bodies elect leadership using first-past-the-post systems. For example, we ask each member to vote for three of the contenders to fill three positions on an International Committee. If 55% of the members vote with the same values, all three winners will be their choices. With an STV system, we could have international committees that are more representative of the membership. Alternatively, we could have regional representation which would disturb the dominance of over-represented regions by giving more power to the votes of underrepresented regions. In this case questions remain: how to divide the regions and are regional distinctions the most important diversity? It is not obvious which is the better system, an STV approach or regional representation. However, it is not ethical to hold onto the worst system just to avoid the difficult questions associated with the alternatives. The ethics are underscored when we remember that members of dominant groups rarely see the need to disturb the systems that established and maintain their power.

An important question is how to educate a membership to enable discussion of possible electoral systems, whether it be a membership of a political party or a body of mathematics educators. It reminds me of a wisdom-saying regarding tree-planting: The best time to plant a tree is fifty years ago, and the second-best time is today. I think that mathematics classrooms are the most appropriate places (times) for students/citizens to experience diverse forms of representation so they contribute to an organization that seeks representative leadership equipped with a robust mathematical foundation for debate about the systems of representation. Prior experience with structures for identifying representation would be mediated ideally by someone with mathematical understanding, someone like a mathematics teacher.

To equip math students for understanding and action, I suggest that mathematics teachers consider constructing situations in which students are subject to mathematical systems used to structure representation so that the students are compelled to debate the structures and mathematically justify their preferences. For example, if a teacher used first-past-the-post voting to decide matters of importance to students, the experience could provoke recognition of unfair representation and dialogue on alternatives. If the teacher “corrected” the problem by using ranked ballots to decide on matters of importance to students, the class could explore different approaches to ranked-ballot counting. A teacher could also ask the class monthly to choose a committee of three or five students to meet regularly to determine matters of importance to students. This could provoke recognition of the challenges of choosing decision makers who represent the members. The teacher could then guide discussion to find ways to *evaluate* the degree to which their elected committee members represent the diversity of the classroom, and to find ways to *construct* fair forms of committee selection.

Subject positions in mathematics

I opened this meditation on subjectivity in mathematics by making a distinction between students being subjects of mathematics and students making mathematics their object of study. As the theorization of subjectivity has suggested, and as my examples of real situations demonstrate, the subject positions available to students are diverse. I argued that we can study mathematics well by putting ourselves into positions of being subject to mathematical structures. That is not the same as being subject to rules of mathematics. In fact, a classroom apparently governed by rules of mathematics may be seen as governed by the teacher who is hiding behind the mathematics.

The role of the mathematics teacher is pivotal in all the classroom contexts discussed here. Each case requires the teacher to cede control. With the two randomization practices, the teacher cedes power to an external (random) force, making students and teacher vulnerable to eventualities that may be challenging. In the election context, I suggested that the teacher cedes power to students. In all these cases, the teacher's authority to cede power is in fact an exercise of power, and the teacher could revoke the new system. Good dialogue among teacher and students can overcome some of the dangers of the new vulnerabilities. As exposed in the consideration of different electoral systems, it is an even greater exercise of power to maintain the status quo, which reifies established power structures. The alternative is to play with structures of power and to discuss the mathematics and related social impacts of these structures. This play can empower students as wary citizens capable of designing and promoting alternative systems.

I invite us all to consider yet other ways in which we can subject ourselves and our students to mathematics.

Mathematics education research investigating positioning and identity is making it increasingly clear that there are various subject positions available to students and teachers in mathematics classrooms. The trend in this research is to identify the relationships amongst the people in mathematics classrooms. Beth Herbel-Eisenmann and I contrasted such a focus on relationships to the early work on positioning in mathematics which tended to focus on how students feel about mathematics (Wagner & Herbel-Eisenmann, 2009). Now I am turning attention again to relationships with mathematics but in a way that exposes how mathematical systems can be and are used for power—both for exercising power and for disbursing power. I suggest that responsible mathematics education would equip students with mathematical understanding of systems of power.

References

Borba, M. & Skovsmose, O. (1997). The ideology of certainty in mathematics education. *For the Learning of Mathematics, 17*(3), 17–23.

Brown, T. (2008). Lacan, subjectivity and the task of mathematics education research. *Educational Studies in Mathematics, 68*(3), 227-245.

Chazan, D. & Ball, D. (1999). Beyond being told not to tell. *For the Learning of Mathematics, 19*(2), 2–10.

De Freitas, E., & Sinclair, N. (2012). Diagram, gesture, agency: Theorizing embodiment in the mathematics classroom. *Educational Studies in Mathematics, 80*(1–2), 133–152.

Fischbein, E. (1975). *The intuitive sources of probabilistic thinking in children*. Dordrecht: Reidel.

Gramsci, A. (1971). *Selections from the prison notebooks*. (eds. Q. Hoare & G. Nowell-Smith). Lawrence and Wishart.

Herbst, P., & Chazan, D. (2003). Exploring the practical rationality of mathematics teaching through conversations about videotaped episodes: The case of engaging students in proving. *For the Learning of Mathematics, 23*(1), 2–14.

Houssart, J. (2001). Rival classroom discourses and inquiry mathematics: ‘The whisperers’. *For the Learning of Mathematics, 21*(3), 2–8.

Klarreich, E. (2017). How to quantify (and fight) gerrymandering. *Quanta Magazine*, April 4. https://www.quantamagazine.org/the-mathematics-behind-gerrymandering-20170404/

Wagner D. & Herbel-Eisenmann, B. (2009). Re-mythologizing mathematics through attention to classroom positioning. *Educational Studies in Mathematics, 72*(1), 1–15.