Positioning in mathematics education: Revelations on an imported theory

Beth A. Herbel-EisenmannDavid WagnerKate R. JohnsonMichigan State University
bhe@msu.eduUniversity of New Brunswick
dwagner@unb.caBrigham Young University
johnson@mathed.byu.edu

Heejoo Suh Michigan State University suhhj@msu.edu Hanna Figueras Michigan State University figuera1@msu.edu

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Abstract: We develop theory within the field of mathematics education based on analysis of an imported theory—positioning theory—and the way it is used in the field. After summarizing positioning theory, we identify some conceptual fuzziness, particularly in core terms 'positioning' and 'storyline.' We offer Lemke's idea of timescales as a way to refine the theory. We then use the refined theory to analyze strong examples from mathematics education literature as a source of insight into how this theory is being and could be used in the field. We identify the need to be clear about scale in describing positioning and storyline, to recognize that multiple storylines and positionings are at play in any interaction, to be specific about the role of communication acts in development of positioning and storyline, and to differentiate among different kinds of positioning. We claim that attention to these issues will help researchers recognize narratives and relationships at play that may be outside their expectation and also underpin stronger warranted claims.

In a little over a decade, mathematics education researchers' attention to the idea of "positioning" has grown, both conceptually and empirically.¹ As we have contributed to this line work, we have found positioning theory to be challenging to apply. We have also noticed that the nature of the positionings mathematics education researchers identified in this literature have varied immensely. Thus, here we develop this imported theory of positioning within the field of mathematics education by examining the way it is used in the field. Our examination of positioning theory and its use in mathematics education literature identified problems with the source theory itself and problems with the way mathematics educators are using the theory (e.g., using it only partially). We contend that, by clarifying some aspects of the use of positioning theory and identify the questions that are not being addressed.

¹ To get a sense of the range of this kind of work, see, for example, Arkoudis & Love (2008), Engle & Conant (2002), Enyedy et al. (2008), Gresalfi & Cobb (2006), Hand, (2012), Herbel-Eisenmann, Wagner, & Cortes (2010), Ju & Kwon (2007), Mesa & Chang (2010), Turner, Domínguez, Maldonado, & Empson (2013) and Wood (2013).

Mathematics education researchers sometimes identify positionings at the interpersonal or classroom level. For example, Esmonde (2009) identified students positioned as "expert" or "not expert" based on how students interacted in small groups. By contrast, sometimes authors identify connections to relatively widespread stories. Esmonde, for example, has also suggested that positionings could occur at broader scales like "socially constructed norms of race, gender, socioeconomic status, and a host of other social categories" (p. 251). The range of scales of the identified *positions* that we have seen in the literature has prompted us to examine further how mathematics education researchers have been writing about it in their work, and how they have been using core ideas of positioning theory. This investigation has led us back to closely examining positioning theory itself, too. Our goal is to suggest some ways to make mathematics education's application of positioning theory more clear. Attention to relationships between people is imperative for understanding mathematics education.

The word 'positioning' has been used powerfully to describe how people experience their interactions with others, even before the term was theorized. Rom Harré, Luk van Langenhove, and their colleagues' (e.g., Harré & van Langenhove, 1999) theorization of positioning has clarified some of the fuzziness in this way of conceptualizing interactions: they connected this use of 'positioning' to existing theories of human interaction and narrowed attention to the moment of interaction to emphasize possibilities for new forms of interaction. Though this focus has increased the power of the use of positioning, we have found that the focus is also problematic because it has not explained well the sources from which people as they engage in acts of positioning. This fuzziness complicates the relationship between two of the three core components of positioning theory – positioning and storylines. These two ideas are somewhat unclear in the way they are explained in positioning theory and the way they are operationalized in both positioning theory and mathematics education research.

We begin with a summary of Harré, van Langhove, and colleagues' positioning theory in order to set the stage for the reader. We then explain the conceptual fuzziness we see in two of the core terms imperative to the theory: "positioning" and "storyline". We offer Lemke's (2000) idea of timescales as a way to refine the theory, in relationship to the conceptual fuzziness. We use ideas from the refined theory to analyze strong examples from mathematics education literature as a source of insight into how this theory is being used and suggest how this work reflects some of the conceptual fuzziness we described. Finally, we provide further rationale for attending to the issues with how positioning theory has been used in mathematics education.

1 Summarizing positioning theory

In order to explicate positioning theory and its key components, we draw on a range of articles and book chapters written by Rom Harré and his colleagues. We begin with chapters from two edited books, *Positioning theory: Moral contexts of intentional* (Harré & van Langenhove, 1999) and *The self and others: Positioning individuals and groups in personal, political, and cultural contexts* (Harré & Moghaddam, 2003), because most authors in mathematics education who use positioning theory cite the opening chapters (Harré & van Langenhove, 1999; van Langenhove & Harré, 1999; Davies & Harré, 1999; Harré &

Moghaddam, 2003) from these books². We also include an article written by Harré & Slocum (2003), because some of the ideas in this article are more clearly articulated than in the books, as well as some very recent pieces by Harré and colleagues (Moghaddam, Harré, & Lee, 2008; Harré, Moghaddam, Pilkerton Cairnie, Rothbart, & Sabat, 2009; Harré, 2012) because they further clarify some of the ideas.

Positioning theory is the "study of local moral orders" as continually fluctuating patterns of "mutual and contestable rights and obligations of speaking and acting" (Harré & van Langenhove, 1999, p. 1). It does not assume, however, that everyone in an interaction has equal access to rights and duties to perform any action in a moment with the particular people with whom they are interacting (Harré, 2012). Harré & Moghaddam (2003) explained that positioning theory can be traced back to Wundt's 'second psychology,' which recognized "that certain types of psychological phenomena, related to language, culture, and collective processes generally, could not be adequately studied using laboratory techniques" (Harré & Moghaddam, 2003, p. 2). Positioning theory is framed in social constructionism and discourse theories (Harré and van Langenhove, 1999), draws on parallel ideas from feminist poststructuralist studies (Davies and Harré, 1999), and relates closely to cultural and discursive psychology (Harré, 2012).

Positioning theory takes as its starting point the constant flow of everyday life, which is segmented into episodes through discourse. Episodes are "any sequence of happenings in which human beings engage which has some principle of unity" (Harré & van Langenhove, 1999, p. 4). Episodes include participants' visible behaviors, thoughts, feelings, intentions, plans, etc. and are defined by their participants and at the same time influence what participants do and say. Discourse is taken to be "an institutionalized use of language and language-like sign systems" (Davies & Harré, 1999, p. 34); language is taken to exist only as concrete occasions of language in use, which is an immanentist view rather than a transcendentalist view of language. As Davies and Harré pointed out:

According to the immanentist point of view there are only actual conversations, past and present. Similarities between various conversations are to be explained by reference only to whatever concretely has happened before, and to human memories of it, which form both the personal and the cultural resources for speakers to draw upon in constructing the present moment. ... It is the actual conversations, which have already occurred, that are the archetypes of current conversations. We remember what we and others have said and done, what we believe or were told that they have said and done, where it was wrong and where it was right. (p. 33)

These authors suggested that an appropriate analytic concept needed to be selected that would "serve to reveal conversation as a structured set of speech-acts, that is, as sayings and doings of types defined by reference to their social (illocutionary) force" (p. 34). Speech-actions are the words or actions one speaks/takes; speech-acts are the meaning that those words/actions have for participants. For example, an utterance like, "Oh, excuse me for forgetting that," could serve as an apology, as an act of deference, or as an act of condescension in different contexts. Davies and Harré (1999) depart from Searle's (1979) version of speech act theory, which sees the

² There are a few earlier references used by mathematics education researchers (e.g., Harré & Gillett, 1994; Harré & van Langenhove, 1991; and van Langenhove & Harré, 1994), but we did not include those here because similar ideas were articulated in these more recent pieces.

meaning related to the intention of the speaker. Instead, they see conversations as unfolding through the joint action of *all participants* as they try to make their own and each other's actions socially determinate. "This way of thinking about speech-acts allows for there to be multiple speech-acts accomplished in any one saying and for any speech-act hearing to remain essentially defeasible" (Davies & Harré, 1999, p. 34). Later work suggested that paralinguistic aspects of contributions like gestures (Harré, 2012) and physical positions and stances (Moghaddam, Harré, & Lee; 2008) also contribute to the interpretations of the speech-action. Thus, we have begun to refer to these as "communication acts" to recognize that social force can be determined by more than just speech.

In order to analyze communication acts in terms of positioning theory, two other core ideas need to be considered as part of the conceptual and methodological framework: storyline and position/positioning.³ Because every utterance can be used to perform several different functions, which one it is will depend in part on which *storyline* speakers take to be in use; any version of what people take to be a determinate speech act is "always open to further negotiation as to what the actual act (if there is such a thing) is" (Davies & Harré, 1999, p. 40). The social meaning of what has been said also depends upon the *positioning* of speakers, which is itself a product of the social force a communication action is taken 'to have'. Because these two additional ideas are necessary to using positioning theory to analyze data, we explain storyline and position/positioning further.

Episodes tend to follow already established patterns of development, which these authors call storylines. *Storylines* are the ongoing repertoires that are already shared culturally or they can be invented as participants interact. Harré (2012) pointed out that storylines depend on the principle that strips of life that are "lived stories for which told stories already exist" (p. 198). Some instances of storylines the authors identified include, for example, "David and Goliath" and "doctor and patient" (Harré & Moghaddam, 2003), instruction and 'hard times' (van Langenhove & Harré, 2003), paternalism, joint adventure, and feminist protest (Davies & Harré, 1999). Davies and Harré (1999) pointed out that the multiple storylines at play

are organized through conversations around various poles, such as events, characters, and moral dilemmas. Cultural stereotypes like nurse/patient, conductor/orchestra, mother/son may be called on as a resource. It is important to remember that these cultural resources may be understood differently by different people. (p. 39)

The names of these storylines indicate what is expected in the episode and encompass the conventions within which to make sense of the events that have been recorded and to express them in a narrative. Harré (2012) noted that storylines can be explicit or implicit and suggested that narratological analysis can help to reveal implicit storylines (p. 198). Explicit storylines, on the other hand, are exemplified in the playing out of structures like ceremonies, rule-bound games, or routines in church. In this case, the positions have been previously decided, and there are ways in which the positions come to be occupied by particular actors. Every storyline, however, incorporates particular kinds of *positions* that relate the participants in various ways.

A position "is a complex cluster of generic personal attributes, structured in various ways, which impinges on the possibilities of interpersonal, intergroup and even intrapersonal action

 $^{^{3}}$ We say more about these two terms in the next section of the paper.

through some assignment of such rights, duties and obligations to an individual as are sustained by the cluster" (Harré & van Langenhove, 1999, p. 1). Position is a metaphoric concept through reference to which a person's 'moral' and personal attributes as a speaker are fully collected. Davies and Harré (1999) described positioning as the discursive process in which people use action and speech to arrange social structures through locating people in conversations "as observably and subjectively coherent participants in jointly produced storylines" (p. 37).

In contrast to the relatively static concept of "role" for understanding human interaction, the idea of positioning was offered to recognize the flexible nature of interactions, which are continually negotiated either explicitly or implicitly. Davies and Harré (1999) stated that role is a transcendentalist concept; position was seen as an immanentist replacement to role and ideas like it. Although authors in the earlier pieces focused on distinguishing positions and roles, in later articles, this distinction was described as occurring along a spectrum. Moghaddam, Harré, and Lee (2008) stated that assignments of rights and duties, as they endure into longer obligations can become a role or are the "birth place" of a role. Harré (2012) also seemed to support this view when he wrote that long-term positions come "close to" the concept of role. Participants may see positions in a conversation in terms of known 'roles' (actual or metaphorical) or characters in shared storylines. Positions may also be very temporary and involve changes in power, access, or blocking of access, to certain aspects of claimed or preferred identity, and so on. A position is linked to the kinds of acts that a person in that position can be "seen" or "heard" to perform. Positions are reciprocal, in the sense that when one positions oneself, others in the interaction are also positioned.

Van Langenhove and Harré (1999) describe many different modes of positioning that can occur, including, for example, first order, second order, and third order positioning, moral and personal positioning, performative and accountive positioning, and tacit and intentional positioning. Although we do not go into detail about these here because they are not central to the rest of the article, these modes of positioning are offered as more specific tools for analyzing these discursive practices.

Although we have explained speech acts, storylines, and positions in separate parts, these parts of the triad are mutually determining. These interconnections especially can be seen in the following quote from Harré & Slocum (2003):

Positions constrain what one may meaningfully say and do. With every position goes a story line. In this way, positioning may diminish the domain of what one *does* out of the possibilities of what one *can* do. We have yet to look closely at the relation of positions and story lines to the meanings that are given to what people say and do. If we think of an utterance as the speaking or writing of a string of words, we can see that in many cases uttering such words is the performance of a social act. Such acts as promising, abusing, thanking, and pleading can have profound and fateful consequences in one's life. Now comes the complication. The very same utterance, the same words, can be heard as the performing of different social acts. ... The third component in the positioning triangle is the story line. Positioned in some given way, a person may be more or less tightly constrained as to what story line it is possible, proper, or even necessary to be living out. (pp. 106-107).

The authors of these positioning articles often depict the positioning triad of speech acts, positionings, and storylines with a triangle. Various versions of this triangle exist, sometimes

placing the three elements at the "corners" (e.g., Harré, 2012; van Langenhove & Harré, 1999) instead of on the edges of the triangle, sometimes referring to "social force of" instead of "speech act" (e.g., van Langenhove & Harré, 1999), and sometimes referring to "rights and duties" (Harré, 2012) instead of "position" or "positioning". Moghaddam, Harré, and Lee (2008) even stated that there is a "possible fourth vertex, the physical positions and stances of the actors" (p. 12). In our revision of the theory we will suggest an alternative image.

2 Critical evaluation of positioning theory

In the previous section, we summarized the basis of positioning theory, its three key components, and how these components exist as mutually determining structures. We emphasize again that we see positioning theory as a powerful theory for analyzing interactions, yet we recognize that all theory is in progress and needs to develop over time. In this section, we highlight aspects of how positioning and storyline are described, which we see as contributing to some of the conceptual fuzziness we have experienced in trying to use this theory. We also identify issues that are cross cutting and that impact the relationships between and among the core components of positioning theory. In some cases, we draw on Wagner & Herbel-Eisenmann (2009)⁴ to highlight some of the issues.

2.1 Positioning

We begin with some aspects that affect the clarity of the idea of positioning. First, one core component (positioning) is used to name the whole theory. Such a move to name an entire theory by one of its parts immediately confuses the part-whole relationship and unintentionally foregrounds one part of the theory. This kind of unintentional foregrounding might impact the use of future work by drawing attention to some parts of the theory (which reference particular concepts) over others.

We find another potential confusion that we alluded to earlier in our use of position/positioning when we examine the use of both "position" and "positioning" by the authors. Although we could not find instances where authors explicitly stated the similarities and differences in these ideas, Harré (2012) alludes to the difference recently when he wrote:

a *position* is a cluster of beliefs with respect to the rights and duties of the members of a group of people to act in certain ways. [...] Because positions are ephemeral compared with roles, the focus of research interest must include the social/cognitive processes by which positions are established. This dynamics is *positioning* – that is, processes by which rights and duties are assigned, ascribed, or appropriated and resisted, rejected, or repudiated. (p. 196, emphasis added)

In this quotation, it appears that 'position' is a noun (a "cluster of beliefs") and that "positioning" is a verb (a "process"). Yet, position and positioning are used in these authors' work as both nouns and verbs. Positioning is also used as an adjective – for example, there are "positioning acts." Additionally, the word positioning is often modified by an adjective, for example, "accountive positioning" or "intentional positioning," but the word position is not. The

⁴ Because some of our references are authored by a subset of the people who co-authored this article, we use "we" to refer to all of the current authors and cite our earlier work with the relevant author names.

grammatical role a word plays in a theory relates to how it is used in analysis because it shapes and bounds the ways in which interactions are interpreted. For example, Wagner and Herbel-Eisenmann (2009) have advocated that authors more often use 'position' and 'positioning' as verbs because nouns relate more to attributes and are stable, rather than acts that are fluid and changing (p. 9).

Our argument that nouns index attributes relates to another issue we have found with position, as it is contrasted with role. As pointed out in the previous section, early work on positioning theory clearly demarcated position as being different from role, an important distinction grounded in the focus on immanence. Yet, the word 'role' appeared in the analysis of positionings. As we noted in the previous section of this article, however, later publications state that recurring positions can become crystallized as roles and that one of the types of positionings relies heavily on the use of roles in interactions (e.g., Moghaddam, Harré, and Lee, 2008).

Positions are defined in terms of rights (what others must do for me) and duties (what I must do for others) (Moghaddam, Harré, & Lee, 2008). This particular aspect of the definition, however, is not emphasized much in the earlier pieces most often cited in mathematics education research. In a recent article, Harré (2012) provided an elaborate expansion of rights and duties as the centerpiece to understanding positions and even replaced the word "position/positioning" with the phrase "rights and duties" when explaining the triad of mutually determining components of the theory.

Harré (2012) also highlighted the two senses of a position he felt were not explained well and included only one of the senses of a position in terms of rights and duties: position was "as the *attributes of a person or group* relevant to positioning and, in the other sense, as an attribution of rights and duties" (p. 191, italics added). We can see these distinctions in their earlier publications. For example, van Langenhove and Harré (1999) offered an interpretation of a transcript from a dinner party and suggested that the participants were, in one part of the interaction, positioned as "teacher" and "learner" and later as "martyr" and "friend" (see p. 18). We see these types of positioning as being relatively metaphorical positioning because the interaction of interest was linked to some other type of interaction that was invoked in the interaction. Other authors described positioning without connecting these descriptions to interactions or roles outside the interaction of interest. For instance, van Langenhove and Harré (1999) later made up an example in which Jones said to Smith: 'Please, iron my shirts' and identified the positioning as "Jones as somebody who has the moral right (or as someone who thinks he has the moral right) to command Smith, and Smith as someone who can by commanded by Jones" (p. 20). In this case, positioning oneself as someone who had the "right to command" can be seen directly in the interaction; it was not metaphorical in the same sense, which could connect this interaction to a similar interaction in another context. We will refer to this as 'personal positioning' as opposed to 'metaphorical positioning'.

2.2 Storylines

As we read across the set of positioning theory publications, we anecdotally note that there is less text about storylines than text that focuses on positioning. As a result, there is less information about storylines. Wagner & Herbel-Eisenmann (2009) pointed out that: a) these authors also used multiple referents to refer to storylines (e.g., narrative, narrative convention,

grand narrative, and story episode), b) there was not a way of establishing a correct storyline in a situation because each participant's perspective might differ; and c) (like positions) there can be multiple storylines at play in any moment. These reportings have been highlighted further in our revisiting of this literature. For instance, in relationship to point a), Harré and van Langenhove call these storylines, narrative conventions and mutually agreed upon contexts, while Harré and Moghaddam (2003) stated that a storyline is expressible "as a loose cluster of narrative conventions" or "as principles or conventions that are being followed in the unfolding of the episode" (pp. 5-6). In a very recent piece, Harré (2012) highlighted our point c) by identifying six different storylines that he saw at play in a conflict between Georgetown University (GU) and the community around Georgetown. He also stated that some of the storylines (i.e., "Students as Savages" and GU as "Neglectful Parent") were:

embedded within a broader set of discursive conventions... "The American Dream:' Coming to live in Georgetown is the ultimate realization of the American Dream. The story is not supposed to end with the hero surrounded by drunken, dirty, and noisy savages. In this storyline, rights (including the right to position others) are acquired to the extent that one pays taxes, owns property, and steps into the dramaturgy of the American Dream (pp. 201-202).

The fact that multiple storylines exist and may even be embedded in broader sets of discursive conventions leads us to wonder: How does the analyst know at what scale to make a storyline? This question is central to the next section, in which we offer a revision of positioning theory. We note that in a few of these positioning theory publications, the authors noted that narratology is the method one would use to identify storylines, but they do not provide more detail than that.

We note that other authors (e.g., Barton et al., 2012; Richardson Bruna & Vann, 2007) who include the idea of positioning in their work have described such culturally embedded narratives as "figured worlds" (Holland, Lachicotte, Skinner, & Cain, 2001) or "Discourses" (Gee, 2011). Wagner and Herbel-Eisenmann (2009) previously pointed out that storyline bears resemblance to "scripts" in Edwards' (1997) cognition-based description of how people draw on known scripts as resources and noted that Holland, Lachicotte, Skinner, and Cain (2001) credited Harré, in particular, as leading the way in considering the nature of self in relation to others and noted connections between figured worlds and storylines. Though there are distinctions in the way these ideas are conceptualized, the unifying characteristics of these concepts (storylines, figured worlds, Discourses, and scripts) is that they emphasize fluidity and focus on similar kinds of grand narratives, highlighting the broader aspects of interactions upon which people draw when they position themselves and others.

2.3 Cross cutting issues

As we have read across the set of positioning publications, we noted that there was some inconsistency and conceptual fuzziness in terms of how the authors wrote about positions/positioning and storyline. We also noted that there can be multiple positions and storylines at play in any interaction and that authors have identified these at multiple levels. An additional issue that appeared is that we found no evidence of the authors operationalizing either of these ideas: that is, the authors do not say how they knew a position or a storyline when they

saw it in data. Instead, the authors offered many excerpts of data with interpretations and described general processes of analysis.

When the authors in the positioning publications described their more general analytic process, they often suggested analysts should start by identifying the storyline, then the associated positions (see, for example, Davies & Harré, 1999; Harré, 2012; Harré & Moghaddam, 2003; Harré & Slocum, 2003). They suggested that the storylines and positions an analyst identified were always conjectures that could be supported or refuted as the analyst moved through an episode. If contradicting evidence appeared, then the analyst needed to identify a new position or storyline. Positions were negotiated when there was an implicit or explicit refusal of an initial (or first order) positioning. The authors did not, however, articulate how they knew what storyline was at play. They also did not state how they knew what the available positions might be within any given storyline, except that one knew these based on the grand narratives or stories we live by.

As we mentioned earlier, the types of positionings and storylines that have been named occurred at many different scales. For example, some articles treated "gender" as a positioning and others treated "gender" as a storyline. These reflections have led us to search for additional theoretical constructs we can use to identify the scale at which positionings and storylines are interpreted because this is important for making clear at what level we see positionings and storylines occurring. Such identifications can help us consider more deeply what scales people bring forth into their interactions. This kind of clarification, we believe, can support authors in communicating more precisely about the focus of their research. Thus, we suggest that positionings and storylines identified. We explain next how Lemke's (2000) articulation of "timescales" can be useful to identifying these various levels and can afford a more nuanced read of the interconnectivity of the various scales of interaction involved.

3 Revised version of positioning theory

Because multiple storylines co-exist and can be embedded in broader sets of discursive conventions (as acknowledged by Harré (2012)), we introduce Lemke's (2000) notion of timescales to positioning theory and promote this revision of the theory for mathematics education research. Lemke, a physicist and science education researcher, drew on dynamical systems theory to articulate how small timescales might be related to large timescales. Small timescales include classroom events like dialogue, thematic units, and micro- and macro-curriculum genres. These events are realized in classroom processes like exchanges, episodes, lessons, and lesson sequences. Larger timescales include events like the organization level of a teaching unit, the organizational level of institutional planning, the biographical timescale (identity change), and the historical timescale (development of new institutions). These events are realized in processes like a semester/year curriculum, a multi-year curriculum, a lifespan of educational development, and an educational system change.

Lemke also identified much smaller timescales (e.g., neural firings, about 10^{-5} seconds in duration) and much larger timescales (e.g., universal change, about 10^{18} sec), but argued that these extreme levels of timescale do not impact people's lives to the same extent as timescales

we have identified in Table 1, which is based on his table. We focus on the ones that he argued impact individual people's lives, because we also see them as relevant to positionings and storylines. Lemke laid out these timescales in a table that showed the orders of magnitude of timescales (in powers of 10 sec) and their associated events and processes.

Typical Process	Timescale (& Duration)	Reference Events
Utterance	10^{0} -10 ¹ sec	Word, holophrase, short monologue; in context
Exchange	$10^2 \sec$ (secs to mins)	Dialogue; interpersonal relations; developing situation
Episode	$10^{3} \sec (\sim 15 \min)$	Thematic, functional unit; speech genre, educative
Lesson	$10^3 - 10^4$ (hour)	Curriculum genre
Lesson sequence	$10^4 \sec (\sim 2.75 hr)$	Macro curriculum genre
School day	$\frac{10^5}{(\text{day})}$	["seamless day"]
Unit	10^6 sec (11.5 days)	Thematic, functional unit
Semester/year curriculum	10^7 sec (4 months)	Organizational level; unit in next scale
Multi-year curriculum	10 ⁸	Organization level; limit of institutional planning
Lifespan educational development	$(\sim 3.2 \text{ years})$ 10^9 sec $(\sim 32 \text{ years})$ 10^{10}	Biographical timescale; identity change
Educational system change	10^{10} (~320 years)	Historical timescale; new institutions
Worldsystem change	10^{11} (3200 years)	New cultures, languages; limit of historical records
Ecosystem, climate change	$10^{12} \cdot 10^{13}$ (32,000-320,000 yrs)	Last ice age

Note. We use only the first word from Reference Events column when we write about these in subsequent text.

Table 1: Excerpts from Lemke's (2000, p. 277)List of Timescales for Education and Related Processes

Lemke pointed out that at any identified timescale the scale below it (a ten-times shorter span of time) provides opportunity for characteristic patterns to emerge, but that the scale above it (ten times longer) provides some constraints for what might actually happen. As Lemke stated:

Each scale of organization in an ecosocial system is an integration of faster, more local processes (i.e., activities, practices, doings, happenings) into longer-timescale, more

global or extended networks. It is *relative timescale* that determines the probability and intensity of interdependence [...], and it is the circulation through the network of *semiotic artifacts* (i.e., books, buildings, bodies) that enables coordination between processes on radically different timescales (p. 275).

Lemke suggested that it is useful to analyze timescales in terms of the scales above and below the focus to help the researcher interpret what is happening. His reference to semiotic artifacts suggested a way for a larger scale interaction to manifest itself in the moment, which is an important aspect of positioning theory. We add that classroom participants also bring into the moment previous experiences with school interaction patterns and other discourses that may seem relevant to them. The way participants bring experiences into the moment of interaction relates to the concept of identities, but positioning theory diminished the idea of identity until recently (Moghaddam, Harré, & Lee, 2008). To illustrate the way larger timespans intersect with the moment of interaction patterns (about 10^{2} sec) that they have experienced in other classroom practices across lessons (10^{3} sec), lesson sequences (10^{4} sec), and units (10^{6} sec). The larger scale interactions that are brought to bear on the interaction constrain the range of offerings the student might provide.

With positioning theory's focus on interpreting any interaction in the moment, we note that the timescales relate to ranges in numbers of participants as well as ranges of time, thus we will refer to scales instead of timescales. For example, students and teachers see themselves as part of a class that is short-lived and also including a limited number of people, but they also see themselves as part of a school, which has more people and runs over a longer span of time. Other larger spans are at play too.

We suggest here that this idea of scales is useful to authors in mathematics education because they clarify the levels of positionings and storylines that might be drawn on and, thus, increase conceptual clarity. This is especially important in mathematics education, which connects classroom interaction to long-standing and wide-spread discourses, including ones that relate to various traditions of formal mathematics.

Addressing the above critiques of positioning theory, we reconceptualize (in Figure 1) the problematic triangle used in positioning theory to reference the triad of speech act, positioning, and storyline. Instead of relying on the fuzzy categories of positioning and storyline, we draw on Lemke's timescales to conceptualize multiple levels of narrative enacted simultaneously, each in its own storyline. Together, these narratives encompass positioning. Also, we replace 'speech acts' with 'communication acts' to include not only speech but also gestures, physical positions, and stances.

Instead of a static triangle, we represent the dynamic, ongoing nature of positioning in relation to communication acts. The arrows indicate that communication acts within any discourse both influence the identification of positioning and are influenced by such identifications. We describe these acts as discourse choices because, as Kress (1993) pointed out, whenever speakers speak, they are making choices (not necessarily consciously) amongst alternative structure and content. The fluid nature of positioning relates to the reality that actors in an interaction may have different perspectives on relevant storylines and thus constantly negotiate positioning either implicitly or explicitly.

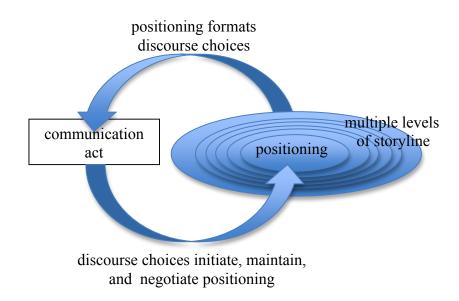


Figure 1: Reconfigured relationship among discourse units and interpretive frames

4 Mathematics education research and positioning theory

As described above, the idea of scales (adapted from Lemke, 2000) offers a useful way to augment positioning theory, particularly in the context of mathematics education research. In this section, we highlight issues that can be seen when the idea of scales is used to consider mathematics education research and we show how using this extension to complement positioning theory can provide greater clarity regarding about what people bring forth into current interactions from past conversations. Specifically, we draw on four different mathematics education studies, which we have chosen for the way they illustrate aspects of the possibilities made available by the specificities we highlight in the theories we propose.

We begin with brief descriptions of the four research articles. Yamakawa, Forman, and Ansell (2009) examined the role of positioning in two third graders' identity construction in a mathematics classroom. The authors showed how the teacher positioned two students, Ophrah and Pulak, differently within a reform⁵ mathematics classroom. Anderson (2009) analyzed classroom interactions to show how larger storylines mediate localized interactions to position students as "kinds" (p. 292). Specifically, she considered how teacher goals and objectives, curricular tools, and classroom interactions collectively impacted student positioning and student opportunities to learn. Herbel-Eisenmann and Wagner (2010)⁶ examined the ways in which frequently-appearing word patterns (called 'lexical bundles') found by computer analysis in a large data set of 148 middle-grades mathematics classroom transcripts related to issues of teacher authority and thus positioned teachers, students, and the discipline of mathematics. Esmonde and

⁵ Here "reform" referred to the use of the ideas from the National Council of Teachers of Mathematics standards documents (e.g., NCTM, 1989; 1991; 2000) in the United States.

⁶ We include a recent article that was co-authored by two of us that was published in *Educational Studies in Mathematics*. Revisiting our own work in this way provides an example of the value of continuous reworking of our theoretical frames.

Langer-Osuna (2013) articulated multiple storylines in a single mathematics classroom (using the construct of figured worlds (Holland, Lachicotte, Skinner, & Cain, 2001)) based on examination of classroom interactions. They described how a particular student in the classroom was positioned and what implications the storylines and positions had for engagement with mathematical ideas and processes. These four articles were exceptional in mathematics education as they featured some analysis that connected to the aspects of theory we promoted above, yet the articles also still illustrate issues in the way positioning theory is used.

4.1 Using "positioning" at many scales

The first issue we highlight is that 'positioning' is used to refer to many different scales across studies. For example, with a cursory reading of Yamakawa, Forman, and Ansell (2009) and Anderson (2009), a reader would notice that both used positioning theory and focused on the positioning of students over time (or student's identity).⁷ In both articles, the authors described the positioning of students as a result of repeated acts of positioning over time; they argued that actions within the classroom informed how students engaged in mathematics teaching and learning (i.e., what positions they came to assume in the space of the classroom or their identities). When we attend to scales of interaction, however, we see some important distinctions.

Yamakawa, Forman, and Ansell (2009) argued that interactions between a teacher and two different students positioned those students differently in the classroom (i.e., that the differential interactions construed different identities). One student was positioned as "conforming to the norms of mathematical proficiency in the reform storyline" (p. 197). The other student was positioned as an advanced mathematics student, even though he did not always conform to the norms of the reform storyline. The authors based their claims about the types of positioning and the storylines at play on two communication acts: the teacher's pronoun use (10°) and revoicing of student utterances (10°) . They also implicated two storylines—reform and conventional mathematics pedagogy. These storylines could relate to a scale 10^{10} because they may occur on a "historical scale" (Lemke, 2000, p. 277) or "educational system change" (Lemke, 2000, p. 277). By analyzing only the classroom practices, however, the authors examined curricular reform at the scale of 10^8 when the teacher tried to enact reform mathematics in her class, which required change at a smaller scale than the larger storyline of educational or school institutional reform. By staying at a classroom practice scale, the analysis might suggest that the storylines of reform and traditional are static when, in fact, they are also ongoing "math wars" (e.g., Apple, 1992; Jackson, 1997a, 1997b; Kilpatrick, 1997; Romberg, 1992; Wu, 1997) that occur at broader scales and influence what happens within the classroom.

Trying to understand how short-term positionings might "stick" over time, Anderson (2009) argued that positionings at micro, meso, and macro levels come together to influence a

⁷ As noted earlier, it was not until recently in positioning theory that identity was discussed in depth (Moghaddam, Harre, & Lee, 2008); however, mathematics education researchers often use positioning theory as a way of addressing identity. The definition of identity, the scale of identity being described, and the relationship between identity and positioning theory are typically not articulated in research. Identities might be viewed as the "positions assumed by people in their interactions with others both in moment-to-moment interaction and as accumulated over time" (Johnson, 2013, p. 22). That is, some authors conceptualize identities as related to specifics contexts and other authors conceptualize identities as related to how positions come to "stick" (Anderson, 2009, p. 291) with people. As a result, identities might be conceptualized at a variety of scales.

person's individual identity (which she characterized as "*kinds* of people" (p. 293)). She suggested that positioning is a:

fruitful construct for understanding and explaining (a) what comes to count as learning relative to teacher formulations of task, (b) the nature and effects of student participation in curricular activities, and (c) the resulting ideologies that mediate opportunities to participate in learning. The construct of positioning is thus important for theorizing the relations between learning and identity development, for developing methodological strategies to examine how students access (or do not access) learning and identity construction resources, and for understanding (and perhaps redirecting) classroom interactions. (p. 292)

As seen in the above quotation, Anderson implicated scales at the exchange level (10^2) , lesson sequence (10^4) , and semester/year curriculum (10^7) in the positionings and suggested that there were implications for the ideologies (10^{11}) that shaped those other scales.

Although we used Yamakawa, Forman, and Ansell (2009) to highlight the variety of scales used to describe positionings, we note that the other two focal articles also include this variation. Herbel-Eisenmann and Wagner (2010) included positionings at the levels of $10^{1}-10^{2}$ (words and interactions among people), 10^{7} (semester/year curriculum), and 10^{9} (identity development). Esmonde and Langer-Osuna (2013) included positionings at the levels of 10^{2} (exchanges) and 10^{9} (identities).

Using scales to consider Yamakawa, Forman, and Ansell's article and Anderson's article highlights distinctions despite the shared use of positioning theory. Anderson drew attention to the ways in which ideologies (10^{11}) shape the exchange level (10^2) . In the classroom she observed, the teacher commented that the focal student had a learning disability. Anderson noted that this type of institutional label and associated beliefs might have affected the way the teacher and students in the classroom interacted with this student. She described how the student's classmates ignored or joked about his contributions and constructed him as someone who could not articulate his reasoning. She highlighted how the ideologies (10^{11}) influenced the interactions among students and the teacher in the class (10^2) . By contrast, Yamakawa, Forman, and Ansell unpacked the relationships among what the teacher said $(10^{0}-10^{1})$, what the students did $(10^{2}-10^{1})$ 10^3), and the positions those actions informed (10^9). Specifically, the authors described how the teacher did not revoice one of the students, while revoicing another regularly. These articles were similar because they focused on characterizing who students become in the classroom and their access to mathematical learning. Yet, the analyses and claims in the two articles were quite different with respect to scales. If the authors located their positionings with respect to scales, it would be easier for readers to see precisely how the arguments differed. In addition to the fuzziness associated with writing about positionings from a variety of scales, we noted three other issues with the use of positioning theory in mathematics education research, which we focus on in the following three sections.

4.2 Singular claims for positions and storylines

The second issue related to authors' tendency to identify positions and associated rights and duties in a singular way. As noted earlier, Wagner & Herbel-Eisenmann (2009) also raised this issue previously. In positioning theory, Harré and his colleagues argued that multiple perspectives on a singular interaction might result in several interpretations of positions. In mathematics education research, however, it is the norm for authors to state a single position for each person participating in an interaction, usually directed by a single storyline (which is sometimes made explicit but is most often implied).

In Yamakawa, Ansell, and Forman (2009), the authors analyzed several episodes of classroom interaction. Although they acknowledged some variation in the positioning of both focus students over time, they identified only one position in each of those moments. For example, the authors analyzed an interaction between the teacher and a student, Ophrah, in which the teacher used specific language to encourage other students to pay attention when Ophrah offered an idea to the class. The authors stated, "Mrs. Porter's suggestion to Ophrah's classmates [...] positioned her as providing useful information" (p. 188). The authors acknowledged that Ophrah's contribution, which was the first one offered in the episode, "showed her willingness to reflectively position herself as an eager student" (p. 188). This identified position represented only one interpretation of the interaction. Perhaps Ophrah or other students would interpret the positioning differently. Similarly, the authors identified this position as occurring within a reform mathematics storyline: "Thus, this exchange indicated Ophrah's conformity to the reform storyline in which students actively engage in communicating their solution strategies to other members of the community" (p. 188). We doubt if everyone in the class would associate this positioning with this storyline. Perhaps Ophrah's quick volunteering could be situated in a gendered storyline. Perhaps the teacher's pedagogy is motivated by a belief that girls should be respected as people who know mathematics, despite their historical marginalization in mathematics (i.e., a storyline of resistance to patriarchal discourse). Instead of being positioned as "providing useful information" (p. 188) in the storyline of reform mathematics, Ophrah might instead be positioned as someone whose knowledge should be respected in this alternative storyline. Readers do not have enough information to suggest alternative positionings, but they might be wary of such univocal interpretations. The same singularity of positions and storylines was found in the other three articles we focused on for this section-although we acknowledge (as in the Yamakawa, Forman, and Ansell article highlighted here) that authors do sometimes provide varied positionings across localized, individual interactions. Authors tended to make overarching singular claims about positions and storylines. The storylines researchers privilege affect which data becomes important, and the way we present positioning and storylines impacts the uptake of the research, thus it is important to be diligent about acknowledging other possibilities, and to actively look for them in analysis.

4.3 Lack of attention to communication acts and storylines

A third issue we identified in mathematics education research related to the use of positioning theory is that communication acts and/or storylines are not always prominent or explicitly addressed, despite being mutually constituting in positioning theory. In relation to the spans discussed above, this point relates to the connection among the utterances in the 10^{0} to 10^{3} scales to larger scales. There are many examples of this kind of missed opportunity in mathematics education literature—we suspect this might be related to the issues related to positioning theory we described earlier (e.g., naming the theory using one component of the theory, less descriptive text about communication acts and storylines). Here we do not focus on articles that missed opportunities to discuss communication acts and storylines. Rather, we illustrate the importance of this issue with two examples in which communication acts and

storylines were specifically addressed in order to show what might be gained when greater attention is paid to these aspects of positioning theory.

Herbel-Eisenmann and Wagner (2010) analyzed a large corpus of transcript from mathematics classroom for frequently appearing word sets (called 'lexical bundles') and connected these lexical bundles to storylines related to teacher authority in the classroom with associated positions for the teacher, students, and the discipline of mathematics. They examined words used at the utterance level (10^1) , phrases such as "I want you to" and "we are going to", within their contexts at the exchange level (10^2) , characterized by Lemke (2000) as "dialogue; interpersonal relations; developing situation" (p. 277). The dynamics of authority in this narrow band of scales $(10^{1}-10^{2})$ could be considered in larger scales, for example, in relation to teachers' evaluation of students at the semester level (10^7) , in students' engagement with mathematics throughout their lives (10^9) , and in other storylines that are longer and larger yet in scope. This kind of specific attention to communication acts and their connections or relationships to the larger storylines is an important task for mathematics education researchers to undertake when considering positioning theory. Although the attention to communication acts and the careful unpacking of those moments of interaction is worthwhile, a close analysis of this article identified missed opportunities to draw connections among the identified communication acts, positions, and storylines. For example, in the results section of the article, the terms "storyline" and "positioning" were not used as prominently as in earlier parts of the article that framed the analysis. Thus, it was less apparent which aspects of the discourse were positionings and which were storylines by the time we got to the end of the article. As the article focused on obligation and authority, the authors turned attention to obligation and authority often with no explicit naming of these as positionings or storylines. The authors could have identified the way these communication acts were influenced by the positions (rights and duties) within storylines in order to more tightly situate the communication acts within these other concepts.

The Herbel-Eisenmann and Wagner (2011) article foregrounded communication acts while backgrounding attention to storyline. By contrast, Esmonde and Langer-Osuna (2013) had the opposite emphases and serves as a powerful example of what can be gained from considering and elaborating storylines, which are not often represented in mathematics education research. These authors used the construct of figured worlds (Holland et al., 2001), which relate to storylines in the sense of scale. That is, they were grand narratives that inform and constrain interactions and positionings. In particular, Esmonde and Langer-Osuna described the figured worlds of mathematics classrooms and figured worlds of gender and romance and considered how these figured worlds influenced student's individual power and authority. This articulation was important because it provided a deeper understanding of the experiences students brought forth into their classroom interactions and resultant positionings. Esmonde and Langer-Osuna began to articulate how these larger scales came to inform interactions in the classroom.

Just as with Herbel-Eisenmann and Wagner's article, however, the use of scales would have allowed Esmonde and Langer-Osuna to clearly articulate the relationships among the components central to positioning theory. We found it was particularly challenging to determine which scale Esmonde and Langer-Osuna were implicating when they described systems of privilege and oppression as well as power. That is, it seemed clear that systems of privilege and oppression should be at a scale larger than individual identity (10⁹). These larger scales might be

then 10^{10} ("historical scale; new institutions") or 10^{11} ("new cultures, languages; limit of historical records"). Esmonde and Langer-Osuna still make a strong contribution to our understanding of these larger scales in relationship to positioning, even without being clear about these scales. Yet, clarification of the scale would provide a more precise understanding of the scales at play in these positionings and facilitate consideration of the implications of the work.

For example, we think a more precise identification of scale would be important in the consideration of power dynamics in Esmonde and Langer-Osuna's (2013) work. They wrote about the individual power and authority that people enact: "Power is constructed relationally and made visible through interactions" (p. 291). The scale implicated in this description seems to say power was evident in "dialogue, interpersonal relations and developing situation" (10²). Esmonde and Langer-Osuna, however, further elaborated power as such:

Although some markers of social status may cut across a number of figured worlds (e.g., masculinity vs. femininity in the work of Holland & Eisenhart, 1990), the specific meaning of any social act is interpreted within a given figured world. (p. 291)

This quotation seems to suggest a larger scale, perhaps individual identity (10^9) was being implicated. The wide gap between these two possible interpretations of power raised questions for us about which the authors might mean. Esmonde and Langer-Osuna later stated:

Our understanding of power dynamics is related to the idea of identities as subject positions within particular figured worlds (see Esmonde, 2009; Langer-Osuna, 2009). These power dynamics play out at the classroom level, but may be associated with broader systems of power in at least two ways. First, perceived mathematics competence has more social "clout" than some other forms of competence (Cobb & Hodge, 2002; 2011). Second, classroom power dynamics may be influenced by race, gender, language, and other categories of social identity. (pp. 291-292)

The authors seemed to be saying both that power was concomitant with a particular role in a figured world but might also transcend the boundaries of a particular figured world. In this description of power, more questions were raised for us. Specifically, for Esmonde and Langer-Osuna does power and authority "stick" (Anderson, 2009) over time to a person and become a part of their personal narrative or is it socially constructed repeatedly in every new context? In what ways might this difference matter? Using scales would offer a way to make clear the answers to these questions as one could understand if 10^9 or even 10^{10} was the level that Esmonde and Langer-Osuna meant to employ or if they were more focused on only 10^2 or 10^3 for characterizing power dynamics between and among people. The attention to storylines here was a powerful and useful description for mathematics education researchers to consider; just as with the Herbel-Eisenmann and Wagner (2010) article, however, Esmonde and Langer-Osuna's article would benefit from the use of scales to the connections among communication acts, positions, and storylines.

4.4 Lack of distinction of kinds of positioning

The fourth issue we identify in terms of positioning theory is that publications in mathematics education do not seem to capitalize on distinctions among different kinds of positioning. For this issue, we cannot illustrate the value of such distinctions using examples from the literature because this is an area of weakness in the literature. Some researchers describe interaction in terms of metaphorical positioning but most in terms of personal positioning. Distinctions among first, second and third order positioning are not made. Most authors that use positioning focus on second order positioning examples, in which there is tension among participants in relation to negotiation of power. We would benefit from examples of how these power relations might be worked out explicitly (third-order), however, and from understanding the relationship between these tensions and first-order positioning (when everyone seems to agree on their rights and duties).

Scales can help mathematics education researchers to understand and clarify some of the conceptual fuzziness that occurs when importing positioning theory. We have identified four problematic aspects in the work of operationalizing positioning theory in mathematics education research: 1) using "positioning" to refer to more than one scale when considering multiple research articles, 2) identifying singular positions or storylines in a given interaction, 3) not specifically attending to communication acts and storylines and the way they shape positions, and 4) differentiating among different kinds of positioning. Using the idea of scales can address these issues by affording researchers an ability to make their meaning clearer for readers.

5 Discussion and conclusion

In this article, we have identified particular issues with how positioning theory has been articulated by Harré and colleagues and have carefully analyzed four articles in mathematics education to show how four of those issues appear in pieces where positioning theory was used well. In this section, we provide further rationale for attention to each of these four issues.

The first issue we raised is the fact that authors identify positions/positioning across more than one scale within individual pieces and across different pieces. Our point here is not to say that everyone must attend to the same scale in order to identify positioning/positions. Rather, we suggest that authors should identify the scales at which their identified positions occur. Identifying the scales at which positions occur can help to:

a) clarify the construct of "position/positioning" as a field in terms of the range of scales within which people might draw as they position themselves and others;

b) shed light on how the scales of positions/positioning might be similar to and different from those of storylines;

c) better understand what resources people bring forth from past conversations as they interact and position themselves and others; and

d) make warranted claims regarding the implications of our findings.

The first two of these reasons relate to clarifying how we are operationalizing this idea in the field. The second reason relates strongly to our next two issues and has implications for equity. By identifying the scale from which people draw, we might become better versed in understanding what, why, and how people draw from past conversations to engage in acts of positioning of self and others. Such identification could put our field in a better place to support students and teachers in developing supportive learning environments. In terms of the last reason for addressing this issue, we think that the identification of scales can ensure that the implications we state from our research align with our units of analysis. Although it is fine to conjecture implications outside the scope of one's data analysis, attending to the scale of

positionings can help us be explicit that our implications are an extension and not an interpretation of the data itself.

Our second issue, the fact that authors are identifying singular positions or storylines in a given interaction, relates strongly to a point clearly stated in Harré (2012) about the relationship between positions and storylines and one's cultural background and experiences:

A fundamental way in which cultures differ is in the taken-for-granted systems of rights and duties implicit in the way lived storylines unfold in everyday social episodes. (p. 191)

When we analyze data, we can only interpret what we see through our own cultural background and experiences. By not pushing ourselves to try to identify more than one possible position or storyline, we may miss very important ones at play for the participants. As Banks and Banks (1995) pointed out, school is a multicultural encounter with both teachers and students belonging to diverse groups based on age, social class, gender, race, and ethnicity. Research is also a multicultural encounter in which there are similarities and differences between the researchers and the students and teachers whose interactions we are analyzing. In fact, such a point highlights the importance of making apparent our cultural backgrounds and experiences by including more about our own positionality in the research we do (see Foote & Bartell, 2011).

Although our analysis focused primarily on articles that included both positionings and storylines, much of the research that we have read identifies positionings, but the storylines and communication acts that inform these positionings are either underrepresented or missing completely. We encourage authors especially to include more discussion about storylines because, as Esmonde and Langer-Osuna (2013) convincingly showed, storylines have implications for equitable practices in schools and classrooms. If we do not attend to storylines in relationship to positionings, we define the problem as being only interpersonal. Yet, inequities can be perpetuated at the individual, interpersonal, institutional, and societal levels and the past conversations about these levels can be brought forth in interactions. Storylines are important because:

One speaker can position others by adopting a storyline which incorporates a particular interpretation of cultural stereotypes to which they are 'invited' to conform, indeed are required to conform if they are to continue to converse with the first speaker in such a way as to contribute to that person's storyline. Of course, they may not wish to do so for all sorts of reasons. Sometimes they may not contribute because they do not understand what the storyline is meant to be, or they may pursue their own storyline, quite blind to the storyline implicit in the first speaker's utterance, or as an attempt to resist. Or they may conform because they do not define themselves as having a choice, but feel angry or oppressed or affronted to some combination of these. (Davies & Harré, 1999, p. 40)

Thus, the storylines within which acts of positioning occur influence the ways in which others respond to acts of positioning. Moghaddam (1999) clearly illustrated how cultural and societal differences influence positionings. Moghaddam, Taylor and Wright (1993) have argued that any satisfying discussion of positionings must take into account the values, ideals, and storylines of people involved. Focusing primarily on positionings restricts our understandings of how these inequities might be perpetuated by the institution of schooling or the stereotypes or values of a society. For example, we have read other articles that connect classroom positionings to capitalism (Richardson Bruna & Vann, 2007) or to popular culture (Epstein, Mendick, & Moreau,

2010). To be clear, we are not suggesting that all authors must attend to all of these ideas equally in everything they write. Rather, we suggest more attention to storylines can help the field better understand systems beyond interpersonal interactions and how the systems are brought to bear in interactions in mathematics learning contexts. As Morgan (2012) pointed out, keeping our focus on the local level:

reduces the descriptive, explanatory and predictive power of the analysis. The challenge is to connect such classroom-level analyses to a developed understanding of the broader context. By locating the analysis of local phenomena within a macro-level analysis of relevant social structures, it becomes possible to see how hegemonic discourses and the interests of dominant groups shape the pedagogic discourse. This more fundamental insight is necessary if we are to understand the forces with which we are likely to be confronted as we attempt to transform inequitable practices in mathematics classrooms. (p. 192)

In addition to linking the interpersonal with broader structures, we think, as noted in our first discussion point, it is important for authors' to be explicit about what they are attending to, at what scale, and why.

Finally, our fourth issue relates to the fact that Harré and colleagues have identified different kinds of positioning (e.g., first, second and third order positionings, personal and metaphorical) yet much of the literature we have read has not identified the modes to which researchers attend. Our read of the literature in mathematics education that addresses positioning is that they tend to focus on first and second order positionings, but they do not name them as such. First order positioning occurs at the outset of an episode when someone engages in a communication act and thus positions him/herself and (reciprocally) positions others. Second order positioning occurs when someone challenges a positioning because s/he does not agree with it. This kind of positioning draws attention to conflict or a mismatch in meaning. Positions or accompanying storylines are then negotiated either implicitly or explicitly. In fact, authors of the positioning articles have recently turned their attention to how positioning helps to understand conflict (see, for example, Moghaddam, Harré, & Lee, 2008). Given the interest in mathematical argumentation by mathematics educators, a specific focus on second order positioning may be useful to this line of work.

By attending to only these two kinds (although not naming them as such to identify the differences), however, we note that we are not attending to other modes of positioning, for example, third order positioning occurs after an interaction when a participant tells someone else what happened and other participants in the original interaction are not present. A focus on third order positionings, for example, might help us to better understand how teachers and students position others in their descriptions of past events. Such third order positionings can draw attention to the reciprocal first order positionings assumed by the speakers and can be used to identify the storylines teachers and students draw on in their re-tellings. For example, Suh, Theakston Musselman, Herbel-Eisenmann, & Steele (2013) showed that understanding the storylines that inform how teachers talk about students in a study group setting can helped to understand the role of institutional tracking and individual maturation storylines in teacher's re-telling of classroom events. Such recognition of these storylines can be useful for helping teachers reframe the narratives that inform their talk about and work with students:

Different storylines (e.g., considering alternative views of school mathematics or using a "toolbox metaphor" (Parks, 2010)) might disrupt teachers' institutional and cultural storylines and at the same time, increase their awareness of how they might act to support students. (Suh, Theakston Musselman, Herbel-Eisenmann, & Steele, 2013, p. 7)

Positioning theory has much to contribute to mathematics education research. All theories, however, are works in progress. The four issues that we have raised are ones that we believe can be attended to by mathematics education researchers and will, thus, improve the clarity and precision of this line of work. We also think our suggestions can help to apply the theory in ways that attend to issues of equity that may not always be the center of attention.

There is value in identifying a range of storylines that connect with mathematics classrooms (and other learning contexts). This range should include narratives of varying scales and cultural contexts, and also forms of interaction that are similar to interactions in mathematics classrooms, which could be sources of metaphorical positioning. There is value in considering which related narratives we are ignoring and why we are ignoring them. There is also value in *describing* (without metaphor) the interactions in the narratives and classrooms to identify the way obligations are developed and addressed. This important work needs to be done with careful attention to the communication acts in particular situations, to help us understand how the positionings underpin the discourse, how the discourse forms and sustains identified positioning, and how the discourse connects these positionings to narratives larger than the classroom interactions. Identifying these relationships will help us understand the nature of equity, access, power, and privilege, which are concepts that are central to most of the articles using positioning theory. Such equity issues are significant because mathematics teaching, learning, and assessment are linked to people's status and access to resources in society.

Indeed, mathematics discourses are pervasive and powerful, and they are connected to many spheres of interaction (not only mathematics classroom). It behooves us as mathematics education researchers to be clear and careful about how we identify these interrelationships. We claim that this important work is in its infancy in mathematics education research. Thus we honor the scholars who are beginning this work (scholars we constructively critique in this paper) and encourage more scholars to take this work up, both empirically and theoretically.

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