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Identifying authority structures in mathematics classroom discourse—a case of a teacher's early experience in a new context

We explore a conceptual frame for analyzing mathematics classroom discourse to understand the way authority is at work. This case study of a teacher moving from a school where he is known to a new setting offers us the opportunity to explore the use of the conceptual frame as a tool for understanding how language practice and authority relate in a mathematics classroom. This case study illuminates the challenges of establishing disciplinary authority in a new context while also developing the students' sense of authority within the discipline. To analyze the communication in the teacher's grade 12 class in the first school and grade 9 class early in the year at the new school, we use the four categories of positioning drawn from our earlier analysis of pervasive language patterns in mathematics classrooms—personal authority, discourse as authority, discursive inevitability, and personal latitude.

1 Introduction

Mathematics comprises truth claims, which are supposed to be authoritative, yet authority is far from simple in mathematics classrooms. Teachers are expected to have authority and also develop students' sense of authority within the discipline of mathematics. This tension is a challenge for mathematics teachers especially when they are new to a school or in the first days of a course. In this article we explore a conceptual frame for analyzing classroom discourse to understand the way authority is at work. This case study of a teacher moving from a school where he is known to a new setting offers us the opportunity to explore the use of the conceptual frame as a tool for understanding the relationship between language practice and authority relationships in a mathematics classroom.

For the past few years we have worked with teachers to better understand the issues they and their students associate with authority and to consider ways of developing repertoires for handling authority issues. We recorded them in selected classes both early on in our collaboration and also when they wanted to pay attention to an aspect of their practice. One of the teachers, Mark,[1] was an experienced teacher who took a position in a

[1] All names are pseudonyms.

different school in his school district during our collaboration with him due to the closure of his former school. He and we found the new school to be an illuminating context for noticing the way a mathematics teacher structures authority in class because he did not carry authority with him from previous years of teaching. His situation had similarities to a novice mathematics teacher, in that he had to establish credibility among the students in the school, and it was a special case of any teacher's need to establish authority structures in a new course.

This article uses a case study of this teacher's experience to explore the use of a conceptual frame for analyzing classroom discourse to understand the way authority is at work. We use the four categories of positioning drawn from our earlier analysis of pervasive language patterns in mathematics classrooms—*personal authority, discourse as authority, discursive inevitability, and personal latitude* (Herbel-Eisenmann & Wagner, 2010). This earlier piece used an analysis of 148 classroom video transcripts that allowed us to find broad trends across a large data set but with less depth in terms of exploring the categories within a particular context. Here we build on that research by using this conceptual frame to conduct a fine-grained analysis of transcripts from Mark's initial teaching context and then from his first weeks in the new school in order to better understand how issues of authority and positioning play out, and how this conceptual frame helps us to see these authority issues. Mark's comments on the experience connect his intentions with his discourse practices.

2 Authority in mathematics classrooms

Authority is one of the many resources teachers employ for control and has been defined in an educational context as "a social relationship in which some people are granted the legitimacy to lead and others agree to follow" (Pace & Hemmings, 2007, p. 6). This relationship is highly negotiable and students rely on a web of authority relations including friends, family members, and the teacher (Amit & Fried, 2005). Educational research related to teacher authority often makes distinctions between different types of authority (e.g., Amit & Fried, 2005; Pace & Hemmings, 2007). Most relevant here are the distinctions made between being *an* authority because of one's content knowledge and being *in* authority because of one's position (e.g., Skemp, 1979). Being an authority means that one's knowledge is deemed relevant to a situation. Being in authority means one is put into a position of power or responsibility by, for example, one's institutional role. Pace (2003) showed that these kinds of authority become blended as participants interact in classrooms. Oyler (1996) argued against the idea that authority is a scarce resource: "for a teacher to share authority is not like sharing a cookie, where if half is given away, only half is left. Rather, when a teacher shares authority, power is still being deployed and circulating, but perhaps in different—and potentially more covert—ways" (p. 23).

Some of these more covert ways were illuminated in our large-scale quantitative analysis of pervasive language patterns in secondary-level mathematics classrooms (Herbel-Eisenmann & Wagner, 2010). In that study, we used computer software to identify pervasive speech patterns (i.e., "lexical bundles") that corpus linguists argue are subtle enough that even discourse analysts rarely pay attention to them. The majority of the lexical bundles we found belonged to a sub-category called "stance bundles," which communicate "personal feelings, attitudes, value judgments, or assessments" (Biber, Conrad, & Cortes, 2004a, p. 966). Stance bundles can be identified by grammatical features

that index implications for participant positioning and relate to teacher authority. We categorized these stance bundles in terms of the different ways they constructed authority relationships. The names of our categories were *personal authority, demands of the discourse as authority, more subtle discursive authority*, and *personal latitude*. For this article we will simplify the names for our second and third categories, and refer to them as *discourse as authority* and *discursive inevitability* respectively. These four authority structures can and often do co-exist in the same conversation. Indeed, our analysis below will demonstrate that.

Accounts of authority and shifts in authority are common in analysis of reform or investigation-based mathematics teaching. For example, Yackel and Cobb's (1996) description of the development of sociomathematical norms in a classroom noted that students were "accustomed to relying on authority and status to develop rationales" (p. 467). Others have promoted approaches to mathematics teaching that would shift authority structures (e.g., Skovsmose, 2001), sometimes without reference to *authority* per se, as with Hufferd-Ackles, Fuson, and Sherin (2004), who described a trajectory to help teachers shift the source of mathematical ideas in their classrooms. With the shift from "teacher as the source of all math ideas to students' ideas also influencing direction […] math sense becomes the criterion for evaluation" (p. 88).

Because authority works in both explicit and implicit ways, we think it is important to develop conceptual models for authority structures and accompanying tools for identifying these structures. Our proposed conceptual model is grounded from the analysis of our earlier large corpus analysis, which is unique within the literature because of its consideration of very implicit ways authority is construed in classroom discourse. The lexico-grammatical features of the model make identification of authority structures relatively systematic. In this case study we will describe the tools in the model and then explore its use in a context that bears similarity to the literature that often discusses authority.

3 Framework for analyzing aspects of authority in mathematics classrooms

As shown in our quantitative analysis, the most pervasive lexical bundles we found were stance bundles (Herbel-Eisenmann, Wagner, & Cortes, 2010), which relate to teacher authority (Herbel-Eisenmann & Wagner, 2010). Because we apply the set of categories we found there to explore a new set of data, we say more about those categories here.

Of these stance bundles, the most common discourse patterns explicitly called on the teacher's *personal authority* and suggested the expectation that students follow the authority of their teacher. This authority structure was identified by the presence of first- and second-person pronouns together. For example, 'I want you to' and 'I would like you to' have the first person pronoun *I* acting as the subject expressing a desire relating to *you.* The interpersonal positioning suggested in the episodes containing these language patterns had the sense of the teacher acting as a guide to students. In this kind of personal relationship the students fulfill their teachers' wishes and trust that the teachers have their best interest at heart. This relates to the teacher being *in* authority. Teachers are placed in a position of responsibility in the classroom and thus direct what happens there.

This language pattern might be used by teachers, by students talking to teachers, or by students talking among themselves, but the quantitative analysis found it almost exclusively used by teachers. If the teacher directs in this way without giving reasons, it

would be an instance of what Alrø and Skovsmose (2002) called *bureaucratic absolutism*. They likened common classroom relationships to frustrations with bureaucracy—"Good reasons or bad reasons, moral reasons, administrative reasons, logical reasons and other reasons—all appear in the same way" (p. 26). Alrø and Skovsmose, as well as others, have identified instances in which students position themselves as teachers in dialogue amongst students (e.g., p. 41). We note that the *personal authority* grammar is often a feature in such interaction.

Another prevalent authority structure in the mathematics classrooms suggested that the discipline had to be followed, which we called 'demands of the discourse as authority,' and which we refer to as *discourse as authority* here. Language patterns that include combinations like 'we need to' and 'we have to' explicitly identify strong obligations because of the modal verbs *need to* and *have to* (c.f. Morgan, 1998)—the rules must be followed. These rules, which come from outside personal relationships, may be attributed to the discipline of mathematics (or perhaps school mathematics). We refer to this discipline as a discourse. In our elaboration on this stance bundle we have noted the importance of the subject in these sentences. When one says *"we* need to" or "*you* have to," those personal pronouns in mathematics often suggest generalization and not specific people (Herbel-Eisenmann & Wagner, 2010; c.f. Pimm, 1987, Rowland, 1992). We also noted a connection to the use of *they* to refer to a non-specified entity or group who have potentially made decisions about the mathematics students encounter; in many cases, this *they* may refer to the discipline of mathematics or some group taken to be representative of the discipline (c.f. Herbel-Eisenmann, 2009).

The *discourse as authority* structure relates to what Pickering (1995) described as *disciplinary agency*, "that leads us through a series of manipulations within an established conceptual system" (p. 115). He noted that scientists are in a sense "passive in disciplined conceptual practice" (p. 115). Alrø and Skovsmose (2002) said that bureaucratic absolutism is "characterised by the difficulty of getting in contact with the 'real' authority" (p. 26) but they did not say what prevents access. We point out that the discourse comprises a huge collective of people and that the grammar of this authority structure both obscures this source and locates it outside the classroom.

A third authority structure in the mathematics classrooms suggested a discourse that obscured the presence of authority but in which actions were predictable, which we called 'more subtle discursive authority' and refer to as *discursive inevitability* here. This authority structure rests on language practices that suggest inevitability—what matters is not the actual probability of an event but rather the language that *suggests* inevitability. With this structure, there is no explicit reference to obligation, but rather a sense of predetermination. Discourse that includes patterns like 'you are going to' and 'it is going to' suggest that there are no decisions to be made. The upcoming actions or thoughts are inevitable. The authority of the participants in the discourse is not recognized with this kind of inevitability. Thus, like with the previous structure, the authority would seem to rest outside of the context somehow. There is no explicit reference to authority, however.

This authority structure may be a deeper version of the bureaucratic absolutism described by Alrø and Skovsmose (2002) and of the disciplinary agency described by Pickering (1995). It is deeper because the language obscures the presence of an authority even more than other ways of expressing authority. When someone says, "you have to," one is reminded of the presence of a rule and perhaps the people behind the rule, but when

someone says, "you are going to," there is no such reminder. This authority structure may support what Alrø and Skovsmose call the "ideology of certainty" (p. 135). We recognize a connection to Bishop's (1988) identification of values in mathematics, specifically to the value of control: "The 'facts' and algorithms of familiar Mathematics can offer feelings of security and control which are hard to resist" (p. 71). The value of control also relates to the *discourse as authority* structure, but the comforting aspects of security probably align with subtle references to predictability more than explicit obligation.

The fourth pattern we found in the mathematics classrooms suggested *personal latitude*, which recognized that classroom participants could make decisions, and thus had authority. This authority structure was the least common of the four in our quantitative analysis (Herbel-Eisenmann and Wagner, 2010). This pattern was identified most usually by the presence of a question. Our analysis below and literature that categorizes questions tells us, however, that student agency is supported only if a question is one that opens dialogue. The distinction between opening and closing dialogue is theorized by appraisal linguistics (for elaboration on this distinction see Wagner, 2012; Wagner & Herbel-Eisenmann, 2008; Martin & White, 2005). Other forms that identified personal latitude described situations in which someone changed their mind. The key to this authority structure is the acknowledgment that people are making decisions. Changing one's mind means one is making a decision. In the first three authority structures, students and teachers are not being framed as decision-makers, but in this fourth one they are.

This *personal latitude* authority structure relates to what Pickering (1995) called *human agency* as opposed to his *disciplinary agency* described above. In the transcripts used in our previous analysis (Herbel-Eisenmann and Wagner, 2010), most of the instances were cases of teacher agency. We believe that a teacher showing students s/he is making mathematical decisions already opens the door for students to see the possibility for themselves. We, like much literature in mathematics education (e.g. Boaler, 2003), however, would promote practices that develop student agency more explicitly. On the other hand, Schoenfeld (1992), while promoting what he called *internal authority,* pointed out its rarity among students, who have "little idea, much less confidence, that they can serve as arbiters of mathematical correctness, either individually or collectively" (p. 62). Roesken, Hannulua, and Pehkonen (2011) emphasized that mathematics students need a sense of autonomy.

The distinction between personal authority and disciplinary authority can be read in the theorization of positioning theory, particularly in the distinction between *transcendent* and *immanent* factors in social arrangement (Wagner and Herbel-Eisenmann, 2009). The discipline of mathematics is *transcendent* or outside the experience and choices of people participating in classroom discourse. This transcendence is evident in our *discourse as authority* and *discursive inevitability* categories above, and identified by others using different terminology (e.g., Alrø and Skovsmose, 2002; Skovsmose, 2001). By contrast, the *personal authority* and *personal latitude* categories described above identify authorities within the classroom.

4 Context and data for this case study

In 2008, we entered a 3-year collaboration with mathematics teachers in Atlantic Canada who expressed interest in considering the way authority works in their classrooms. After interviewing each teacher at the outset, we recorded 15 consecutive sessions of a

mathematics class they each chose. The group of teachers met with us about once every six weeks during the research. Further classroom recording was done when they wanted to try new things related to authority. In addition to video recording, we used voice recorders to capture more local audio of students' group work. We also interviewed the participant teachers periodically and sometimes interviewed students who were in the classes that were recorded.

Mark, the teacher who we focus on here, had taught mathematics for 4.5 years prior to this study. He was teaching all mathematics courses for grades 9-12 mathematics in a rural high school with about 150 students. Mark chose a grade 12 classroom for observations. The students' families generally had incomes lower than the provincial average, lower yet than the national average. Many parents worked in the forest industry and/or commuted about 1-1.5 hours to a larger centre for work. After the first year of our collaboration, Mark took a position in an urban school with well over 1000 students with more diverse family contexts. Now instead of being the only mathematics teacher in the school, he was one of many. He taught multiple sections of grade 9 mathematics and grade 11 physics. Students did not know him, so he described a sense of having to establish his authority both mathematically and as a teacher who cares for his students. Mark's situation provided a setting in which we could explore the case of how a teacher considers and enacts authority in changing contexts (i.e., from a familiar context where he was comfortable and established in a small school to an unfamiliar context with different demographics in a much larger school) to develop understanding of the way our categories described above can give us insight into the way authority works.

As is common in case study research, the data and analyses were interwoven. We began with conversations with the teachers about authority, were able to observe them teaching, and had continued conversations with them about their considerations. We iteratively sought and discussed the patterns we observed and modified the interview questions and observations as needed (Yin, 2006). For example, we recognized that changing schools could allow particular aspects of authority to surface and thus agreed to observe almost every day as Mark's school year began. We see his situation as an interesting case of a teacher grappling with authority in two different contexts over a period of time.

We present this longitudinal case study in chronological sequence (Yin, 2006). In addition to our descriptions of the changing contexts of Mark's teaching, we analyzed transcripts from his familiar context, teaching a grade 12 mathematics class, and from his first weeks teaching a grade 9 class in the new school. We analyzed those transcripts in terms of the four authority structures identified in the section above. We did more than look for the lexical bundles that helped us identify those four categories. We looked at the grammar for patterns of speech that resembled those lexical bundles and we also looked beyond the grammar for other evidence of the authority structures. Table 1 operationalizes the conceptual frame and guides our analysis of classroom communication using the four authority structures.

Authority Structure	Linguistic Clues	General Indicators of the Structure (that may not involve the particular linguistic clues previously identified)
Personal Authority	• *I* and *you* in the same sentence • Exclusive imperatives • Closed questions • Choral response	Look for other evidence that someone is following the wishes of another for no explicitly given reason.
Discourse as Authority	• Modal verbs suggesting necessity (e.g., *have to*, *need to, must)*	Look for other evidence that certain actions must be done where no person/people are identified as demanding this.
Discursive Inevitability	• *going to*	Look for other evidence that people speak as though they know what will happen without giving reasons why they know.
Personal Latitude	• Open questions • Inclusive imperatives • Verbs that indicate a changed mind (e.g., *was going to*, *could have*) • Constructions that suggest alternative choices (e.g., *If you want, you might want to*)	Look for other evidence that people are aware they or others are making choices.

Table 1. Analytical guide for identifying authority structures

5 Considering authority as context changes

In the following application of our conceptual frame, we first give contextual information drawing on what Mark shared about his thinking about authority at the outset of the research. We then analyze transcripts from each of the two school settings. Finally we give an account of Mark's effort to transform the authority structures in his new setting by explicitly addressing authority issues in conversation with his class.

5.1 Talking with Mark about authority in the familiar context

In the initial interview with Mark, he was asked about his role as a mathematics teacher, to which he replied:

> The students look at you as their sole source of knowledge, very few [take] the initiative to go and find answers on their own. […] Like, if you run through investigations with them, by the time you get to the end they look at you and go, 'Why didn't you just tell us that?' […] They're quite reluctant to accept the authority really. (Mark, first interview)

Mark's characterization of his mathematics classroom authority structure aligned with personal authority. Students relied on him for guidance. He wanted them to "accept

the authority," which would suggest his hope for them to exhibit personal latitude. He did not seem to mention ways in which mathematics as a discipline has a role in the authority relationships in the classroom.

Mark's conceptualization of his classroom discourse was quite focused on authority and was, of course, skewed by participation in this research. When asked more focused questions about authority, Mark's attention moved toward his students working on exercises to reinforce and apply the ideas they learned in their investigations. When asked, "What or whom do your students see as authorities in their classrooms?" he said:

> I don't think they look beyond [us math teachers]. They feel like we should have all the answers. And sometimes they don't realize that sometimes we have to go look for answers as well. So even though we demonstrate that the authority is found in other places, like textbooks and other colleagues and things like that, they still, … they're focused right in on their teacher. Their teacher must have all the knowledge. (Mark, first interview)

It is clear from his unprompted references to the textbook mandated for use in the province's mathematics classrooms, that this textbook was a source of authority for Mark and for students in his classroom. Indeed, he used the textbook every day we observed as a source of investigations and/or a source of practice problems to assign to students.

When asked what would happen if he were to disagree with the textbook, he stated that students would "have a hard time believing me over the textbook." He recalled situations, however, in which he went through answers with his students who were then convinced that there was an error in the textbook. Nevertheless, Mark's focus in this interview somehow switched from developing understanding to "getting answers."

When asked, "How do students know what to do in mathematics?" Mark did not seem to understand the question. Perhaps the idea that students do what their teacher tells them was hegemonic to Mark and, thus, the question did not make sense. When we focused the question by asking about how students decide what to do when addressing a problem, he said, "Some of them that have actually remembered previous teachings will just […] automatically go to the rules they've previously learned." They would look at the examples he gave, but "some will just constantly ask you, 'What do I do now?', 'What do I do now?', 'What do I do now?'" Mark's frustration with students' dependence was palpable.

5.2 Observing Mark teach in the familiar context

In Mark's familiar context, the classroom in which he taught for 5.5 years before changing schools, we found examples of each authority structure. We selected the transcript below for this article because it was typical of what we observed and it includes examples of each authority structure. The structures are co-existent and not straightforward to identify in some cases.

a7	Mark	Okay, so we've been looking at things like going on a trip and calculating things like your average speed between points in the trip, okay? […] In a car, you use your odometer. […] What about your speedometer? What does it measure? What is it telling you about your speed?	
a8	Zach	How fast you're going per hour.	[*simultaneous*]
	Rachel	How many kilometres you're going in an hour.	

a9	Mark	Per hour, okay. But is that actually how many kilometers you're going to travel in an hour?
a10	Zach	No.
a11	Mark	No. Okay, what is your speedometer really telling you?
a12	Alan	How fast you're going.
a13	Mark	Right, it's telling you how fast you're going at that very moment. Okay, so that's the next topic we're leading into. All right, we're going to start looking at instantaneous rates of change.
a14	Lucas	Are these notes?
a15	Mark	Business as usual. Okay, so we're starting exploring, having to find instantaneous rates of change—how fast things are changing at that very moment. [*pause*] So what is it? Technically speaking it is the change in a dependent variable over an infinitely small change in the independent variable, all right. That's the technical words for it. As you move on into grade twelve they'll start speaking about limits.
a16	Connor	We've done those already.
a17	Mark	No not really. So in grade twelve they'll start talking about how the independent variable approaches a particular value. And as we mentioned, instantaneous velocity is an instantaneous rate of change. The instantaneous velocity or the instantaneous speed is what your speedometer measures in your car. In that case, it's a change of displacement over an infinitely small period of time. In other words, right now. [*He hands out a single sheet of graph paper to each student*] All right? So far so good? Okay you don't have to copy this down. Okay so as we mentioned up to this point we're going to calculate average rates of change. Okay, but what we're going to look at today is if we take those two points, okay, and we bring them closer and closer together. We calculated average rates of change over various periods or various intervals, right? But what if we start bringing that interval closer and closer and closer together?
a18	Zach	What happens if they touch?
a19	Mark	What happens if they touch? Then you get an instantaneous rate of change. If we want to find the instantaneous rate of change on a particular graph we can approximate this value by decreasing the interval that concludes this point or it could be the intervals at the lower extreme or the upper extreme of that interval or you could have two points coming closer and closer from either end, okay? So what I need you to do now is just to sketch this graph okay? Just your *y* is equal to *x*-squared graph.
a20	Zach	Do you want us to draw it up all the way to 4.5?
a21	Mark	Sure or you can just fill every two blocks and just go "one, two" all the way up to five. And your *y*-axis should go up to twenty, okay? [*He walks around the classroom checking students' work*.] Okay, try to plot the points okay? So make sure you go over one and go up one. Over two, up four, over three up nine, over four, up sixteen, and that's the last point you can plot when you go up to twenty on your *y*-axis. [*Students work*

		quietly for a few minutes.] Okay, so what we're going to start looking at is we're going to use *x* is equal to 4 and what we're going to eventually try and find the instantaneous rate of change, okay? But to start, I want us to find the average rate of change, okay, from zero over to four. So the interval would go from zero [*He writes on the board*], okay, so in other words we're finding "*f* at four" minus "*f* at zero" over "four minus zero." So "*f* at four" would be? Sixteen okay, "*f* at zero" would be zero and all over [*more writing on the board*] okay? We'll then find the average rate of change from one to four. [*He writes more on the board*.]
a22	Ella	Why did you put the less than equal to sign after the one?
a23	Mark	In here?
a24	Ella	Yeah.
a25	Mark	Because this is the interval where we're going from one up.
a26	Ella	Okay.
a27	Mark	Okay, so what do you think the next interval will be?

In this transcript, there is evidence of personal authority. Considering the grammar that resembles the lexical bundles exemplifying this authority structure, we look for the pronouns *I* and *you* in the same sentence. Here, we find Mark saying, at the end of turn a19, "what *I need you to do* now is…" Also, in the middle of turn a21 he said, "*I want us* to …" His reference to *us* includes the students so he was articulating his expectations for them; this is similar to "I want *you* to find …". In both these cases, students were not given a reason; they were merely expected to sketch the graph because Mark "needs" them to do so. Without a reason, students may have been stumped when it came to making decisions in their work. For example, how should they scale the graph? If they had known the reason for Mark wanting them to draw the graph, they could have thought about how to scale the graph, but because they did not know for what the graph would be used, they would wonder how to set up the graph. And so, a boy asked in response, "Do you want us to draw it up all the way to 4.5?" (turn a18) and Mark responded to this question with even more detail about how to draw the graph, still with no reasons for these detailed instructions.

This pattern of students asking Mark what he wants them to do was prevalent. Even when Mark did not give explicit instructions, it was clear they were relying on his personal authority to tell them what to do. For example in this transcript, in turn a14, someone asked Mark, "Are these notes?" He relied on Mark's authority when deciding what to write and what not to write in his notes.

In this transcript, there was also evidence of the discourse as authority. Mark positioned the authority of the discipline of mathematics as being transcendent, outside the classroom. Considering the grammar that resembles the lexical bundles exemplifying this authority structure, we look for modal verbs that suggest necessity. We find the modal verb structures *have to, need to,* and *should.* In the middle of turn a17, Mark said "you don't *have to* copy this down" and, as noted above, in turn a19, Mark said, "what I need you to do now." In these cases, the necessity points to Mark's immanent authority, not to a transcendent source. In turn a21, Mark points out "your *y*-axes *should* go up to twenty" but without explanation as to why. The only other modal verb structure pointing to necessity in this transcript is *you can*. Mark said, in turn a21, "that's the last point *you can* plot." He told the class that it was impossible to go further. Based on our experiences teaching

mathematics, it seems to us that Mark would have had mathematical reasons for saying what they *should* and *can* do here, although students may have been wondering whether this was merely another instance in which they should follow Mark's authority.

In addition to the modal verbs indicating a disciplinary force that regulates action, we note that Mark marked the discursive power of the discipline by referring to vocabulary definitions coming from outside the classroom: "Technically speaking it is the change in a dependent variable over an infinitely small change in the independent variable" (turn a15). He did not say, however, where he found these definitions. With the absence of personal pronouns here, in juxtaposition with his pervasive use of *we* in many of his other turns, he points to a transcendent discipline. Additionally, we wonder whether directions to "*make sure* you go over one and go up one" might fall into this authority structure because a procedure is being described as if there are no other choices, and no reasons are being provided for why someone might follow this procedure.

This transcript also presents evidence of discursive inevitability. Considering the grammar that resembles the lexical bundles exemplifying this authority structure, we look for the modal verb structure, *going to*, because it suggests knowledge of what will happen. In this case, Mark employed this structure not to claim knowledge of what the mathematics will produce, but rather his knowledge of what he and other teachers would have his students do. He begins in turn a13 by stating, "we're going to start looking at instantaneous rates and changes." In turn a15, he said that in "grade 12, they'll start talking about limits." *They will* apparently refers to the students' teacher in a future grade 12 class (there are two grade 12 mathematics classes for students aiming for university matriculation in the sciences). This reference to *they* is odd because Mark would be their teacher for that class. So, he may have been referring to the textbook or curriculum with his pronoun *they.* This structure continued in turn a17 with the same "they'll start talking" and also a more immanent future—the actions of the class this day—"We're going to calculate" and "we're going to look at." These instances of the discursive inevitability blend with personal authority because Mark's confidence that the students would be doing these things is due to his expectation that they will do what he will ask.

Though the instances of language suggesting discursive inevitability in this transcript are blurred with personal authority language, Mark did employ a more mathematically-focused discursive inevitability later in the same class. Before allowing students to work out a problem he said, "So we're going to get 4188" (turn a74). There was no doubt what would happen, thus the actions of the people in the classroom (including himself) were deemed redundant. In this case the source of Mark's confidence was not his social control. Rather, his knowledge of what would happen was based on the mathematics. 4188 was the only correct result students could get.

Finally, this transcript presents evidence of personal latitude. Considering the grammar that resembles the lexical bundles exemplifying this authority structure, we look for questions that open dialogue, instead of closing it, because such questions invite multiple voices, multiple possibilities and perspectives. We also look for "if you want to" and "was going to." These relate to possible intentions. At the end of turn a17 Mark asked a closed question: "what if we start bringing that interval closer and closer and closer together?" It is a closed question because he has a particular answer in mind. When someone asks, "What if they touch?," however, there is evidence of a classroom expectation that it is permissible for students' mathematical questions to divert Mark's plan. There are

numerous examples of this kind of diversion in this class. In this transcript a further example appeared in turns a22 to a26 when another student asked Mark for clarification. These students demonstrate that they took Mark's discourse as opening dialogue even when the structure of his speech seemed to close it. The feature of Mark's speech that makes this phenomenon clear is his willingness to take up their questions. Yet, although students were expressing personal latitude by raising their own questions, they were still relying on his authority as they looked to him as a representative of the discipline to answer their questions.

Also in this same class session a girl asked Mark if there was an easier way to write the interval $0 < x < 4$. A boy asked if the method being discussed would always give the rate. In the first half hour of class (all whole class discussion) five of the eleven students took initiative to ask questions. Mark set the agenda (following the curriculum) but students exercised their personal latitude by thinking about what eventualities they might face and asking Mark for clarification that might help them face these eventualities.

There are other examples of personal latitude as well. Mark said in turn a19, "*If we want* to find the instantaneous rate of change," recognizing that the class may have an intention to do so, and later in the same turn said "or it *could be*" and "or *if you could have,*" suggesting that he and the students have choices in how to go about their mathematics. Some of these acknowledgements of intention and possibility, however, may have been rhetorical because finding instantaneous rates of change was demanded by the curriculum.

The personal latitude expressed by students in Mark's class could be attributed to various factors. Most importantly, Mark was responsive to their questions and thus encouraged more. Furthermore, intimacy had a chance to develop within the small class that comprised a relatively stable cohort over twelve years and between them and Mark.

5.3 Observing Mark teach in a new context

The circumstances that supported the discourse that Mark and his students negotiated did not follow him to his new school. The students in the class described above had had Mark as a teacher for a few years, and some of them had older siblings and friends that had also had Mark as a teacher. But in the new school, none of his students had previously met or heard of him.

Mark and we agreed that recording the initial classes could be revealing. We all noticed that Mark and his class settled into discourse that was significantly more reliant on him as an authority. Although he felt that he had to establish his mathematical authority he continued to say he desired a situation in which the students would "develop their own authority." Having to adjust to a new large school themselves, these grade 9 students may have felt lost in their first year of high school and thus more reliant on their teacher.

As with the previous school context, each of the four authority structures appeared in this class—personal authority, discourse as authority, discursive inevitability, and personal latitude—but Mark described this group as much more dependent on him. We selected the transcript below for this article because it was typical of what we observed in this classroom and it included examples of each type of structure. It was from the earliest complete transcript we were able to collect due to the time it took to get the students' consent to be recorded. We pick this conversation up where Mark is leading the group through the prime factorization of 72. They have 3 x 3 x 2 x 4 so far.

b134	Mark	In order to perform the prime factorization we have to break it down so that all the factors are prime numbers. So as of right now, we have three of our four numbers are prime numbers, correct? So keep working. So now we have, "Two times what are the factors of four?"
b135	Alexis	Two times two.
b136	Mark	Two times two. Two times two is what the four was. And then we have our times three times three. Of the five factors we have now, how many of them are prime?
b137	Students	All.
b138	Mark	Okay, if we look back over here, "Two times two times two times three times three times three." That's how we get from seventy-two. This is how we perform our prime factorization. Okay. So that's why I was saying it's not expected that you know that this right away is the prime factorization of that.
b139	Simone	Where would we need, where would we use a question like that?
b140	Mark	You are going to use it later on. It makes it very easy later when we are cancelling out or dividing by numbers
b141	Jerry	No, what's a job where we would need
b142	Mark	What job? Uh, not everything we do in math in high school is going to give you, uh, is going to be used in everyday life. Okay. Everyday life you do some adding, subtracting, multiplying and dividing, right? Okay.
b143	Emily	I sleep.
b144	Mark	You sleep. You don't spend any money? Okay, anyway the purpose of our math courses is to give us all the tools that we need, right. So that later on when you decide on a career that you want to do that you have all opportunities open to you.
b145	Kate	What if you want to have nothing to do with math?
b146	Mark	Oh everything has to do with math.
b147	Jordan	What if she wants to work at McDonalds?
b148	Mark	Money, money, money is math, math, math.
b149	Students	[*Many students are talking*.]
b150	Mark	All right. Back to the rules of mathematics. Back to the land of the living. Okay, so let's find factors and prime factorization. Okay. Try this one out on your own. I want you to find all the prime factors of thirty-two. Thirty-two, prime factors of thirty-two. Use your divisibility rules if you're stuck.

In the transcript, there is evidence of personal authority. First, we look for the pronouns *I* and *you* in the same sentence. In turn b150, Mark said "I want you to find all the prime factors of thirty-two." *I want you to* positions the task as being either for his benefit or a relationship of trust in which students following his guidance would yield a beneficial result. Given the conversation that preceded this statement, it is possible that the beneficial result could be just finding the prime factorization, being prepared to use prime factorization when they do things like "cancelling out or dividing by numbers," or possibly even preparing students for being able to do whatever they want to do when they "decide

on a career." Mark gave no reasons for the students to follow his instruction except for identifying this as his wish.

In addition to instances in which the grammar alerts us to a personal authority structure we see that Mark used bald imperatives and questions that close dialogue. In turn b134 he told students to "keep working" and in turn b150 he said, "Try this." These imperatives expected only one course of action, though the verb *try* is relatively invitational. Mark's closed questions were taken as closed by these students, unlike the students in his former environment. When he asked in turn b134, "Two times what are the factors of four?" a student responded with the one expected answer, "Two times two." And when Mark asked in turn b136, "How many of [the factors] are prime?" students responded in chorus with the one expected answer, "All." The choral response is a strong indicator of a personal authority structure because the whole class demonstrates agreement that their role is to follow Mark's wishes. It is also an indicator of discursive inevitability because the choral response recognizes agreement that there is only one possible answer.

In this transcript, there was also evidence of the discourse as authority. The modal verb structure *have to* draws attention in turn b134 to the singular course of action imposed on the teacher and students by the mathematics—"In order to perform the prime factorization *we have to* break it down so that all the factors are prime numbers." Further evidence of the discourse as authority appears in this transcript where Mark noted, "it's not expected that you know this right away." It is unclear who would not expect the students to identify prime factorization immediately without this longer process, but the grammar suggests it would be someone or something outside the classroom context. A more explicit reference to the controlling discipline of mathematics can be found in turn b150 where Mark turns the students' attention away from their concerns with the demand, "Back to the rules of mathematics." In addition to this explicit call to the discipline's authority, he exercised his personal authority to guide/control what happens in his classroom.

This transcript also presents evidence of discursive inevitability. We note the modal verb structure, *going to*, in Mark's response to student questions about relevance. Thus, like in the earlier context described above, Mark did not reference the inevitability of mathematical results. In that context he referenced the inevitability of what would be done later in mathematics classes, but in this new context he was referencing first the inevitability both of future mathematics classroom practices and of the trajectory of the students' life experiences. In turn b140 he said, "You are *going to* use [this skill] later on." When students clarified that they wanted relevance in their lives, not in future mathematics classes, Mark continued with the discursive inevitability structure, saying that their mathematics skill "is *going to* be used in everyday life" (turn b142). He went further, in turn b144 to foresee that students would be deciding on a career—"when you decide on a career"—though he did not use *going to* language structure. These instances of discursive inevitability have the same grammatical structure as "we're going to get 4188" (turn a74 in the earlier episode), but Mark's confidence does not seem to be placed in the same kind of reasoning in this instance.

Finally, this transcript presents evidence of personal latitude, but there are significant differences from the focus of student agency in the previous context. Again, we note questions, which are a mark of personal latitude. In turn b139 a student asked, "Where would we use a question like that?" Mark's response suggests that he took this question to be like the questions he was familiar with in his previous school context. He

seemed to think the student was asking about the application of this skill to further mathematics. The student corrected him in turn b141, "No, what's a job where we would need…" Mark was complicit to the student's expression of personal latitude by responding to the question, and other students participated in this discourse, one with the provocation, "I sleep" (turn b143), and one with the legitimate question about careers that do not need high-level mathematics (turn b145). When this discourse turned into a buzz (turn b149) Mark exercised his personal authority and cut off the students' autonomous questions.

5.4 Re-negotiating authority in a new context

Mark was concerned about the dynamic in his new classroom. After two months of frustration with what he saw as his students' lack of mathematical agency, he chose to set aside time to challenge students with questions about authority. He started a class session telling students about his interest in authority as a research participant. The following excerpt comes from near the beginning of the class session:

c17	Mark	We're looking at [authority] not necessarily the way that you guys probably think of authority. We're not talking about necessarily who's in charge, per se. That kind of authority. Like police kind of authority. Now that does play a little bit of a role in a classroom obviously. But we're looking more at authority as to the holder of knowledge. Who is the holder of knowledge? Am I?
c18	Students	No
c19	Mark	Okay
c20	Girl	It's us.
c21	Mark	Okay. Good. There're lots of sources of authority. Right? If we're talking about mathematical authority, there're lots of sources. Correct? I am, I guess. I consider myself a source of mathematical authority in the classroom. But, I also consider each and every one of you guys a source of mathematical authority. […] the whole idea is to disburse the authority a little bit more so that it's not just one big source, and that's the only place where you can get information, the only place you can think of as being a source of knowledge, a source of information. The idea is to make yourself your own source of authority.

Mark then displayed with his projector 2 + 3 = 5 and 2 + 3 = 7. He asked which expression was true and why. Many students became restless. At first, students said that they knew 2 + 3 was 5 because teachers said so, which suggested their reliance on teachers' personal authority. Eventually a girl explained why it has to be five, demonstrating understanding that the discipline (discourse) can have authority; she grouped two fingers on one hand with three on her other hand and said, "We learned it when we were younger—the counting numbers. We used our hands to count and adding numbers. Through the years you kind of adapt to it being five."

Next, Mark displayed two further equations, 2 + 3 x 5 = 25 and 2 + 3 x 5 = 17. One boy said, "It depends on how you do BEDMAS" (Brackets, Exponents, Division & Multiplication, Addition & Subtraction). Mark revoiced this statement and the class erupted. One voice stood out saying, "If you do it right you get 17, if you do it wrong you get

25." This suggested an appeal to the discipline as authority or discursive inevitability—only one answer is possible—but it was unclear from where this authority comes. When Mark asked who decided on this order of operations the students guessed names: *you* (i.e., Mark), Stephen Hawking, Albert Einstein. The students concluded that the convention was passed down through generations, but were vague about how the convention started. Someone suggested "the beginning of time."

Mark was no longer following a plan and he was speaking about as much as the students (in usual class discussions he spoke much more than the students). Significantly, the students began exercising personal latitude by making demands of him. 31 minutes into the conversation a girl said, "You are asking a hard question. An example would be really helpful." Mark responded with a scenario in a game and another girl interrupted, "No, a real life example." Then Mark started using an example from when he built his deck, but students argued for an example from their real life, not his. When he used the example of choosing mobile phone packages the class was finally content with that example.

When Mark challenged his students with questions about authority, they exercised authority by telling him how they wanted him to teach them. Reflecting on the conversation, one student said to Mark, "You asked all these questions but they didn't have answers." The conversation was about 44 minutes, evidencing the students' interest and Mark's dedication to developing a different authority structure in class.

We were curious about how this exchange would change the classroom dynamic. It was not possible to characterize the class as fitting one authority structure, however, because all four structures appeared in every class, albeit with variations that may only be possible to describe qualitatively. Mark noted that the students began to ask questions after this conversation, which suggested more personal latitude than prior class sessions. They were becoming like the class in his previous school. In a formal presentation to teachers later in the year, Mark characterized these students as "very frustrated," "not engaged in their own learning," and "passive participants," at the beginning of the year and he identified a move to "students questioning," "asking for alternative methods," "demanding explanations," and "giving their own examples of problems they wish to know." The passivity at the beginning of the year suggested a personal authority structure where the students did whatever he said with little question. He also gave an example from five months later: a student asked him to demonstrate a certain kind of problem, and others gave further directions to him about what they wanted demonstrated, and even posed their own problems. He saw this as a shift toward students sharing authority for their own learning. This was a shift to more personal latitude. His descriptions focused on who in the room exercised authority, and did not consider how the discipline of mathematics was also an authority.

6 Reflection

In this case study our conceptual frame helped us see the complexity of authority in mathematics classrooms and to Mark's position as a teacher. The case raises questions for us. First, although the four categories were first uncovered through a large corpus analysis that allowed us to see broad patterns in the stance bundles, we can also see how these categories are useful for unpacking some of the ways in which authority is instantiated in classrooms. We found that it was useful to draw on the grammatical features of the stance

bundles but we also found that there were additional ways the four categories might be instantiated that might be missed with our primary attention to grammatical features.

To continue development of this conceptual frame, we think it would be useful to focus more specifically on one of the categories in a particular case and to explore further how authority is structured in that category, as well as to consider the possibility of additional authority structures in mathematics classrooms beyond the four structures we identified. Delineations such as those in our conceptual frame and further delineations that come from extending it may sometimes over-simplify the complexity of classroom interactions. Yet, we recognize that these delineations provide useful lenses with which teachers could see and talk about their classroom practices and make more purposeful decisions about how they want to negotiate authority with students.

Second, in relation to Mark's positioning in the classroom, we wonder whether these categories might be useful to understanding how authority changes over time not just in different contexts but also in the same context. It would be interesting to see whether different kinds of contexts might matter. For example, in the rural context where Mark had been for years and had extensive knowledge of the families and students, it is possible that personal authority became foregrounded because there was time to establish trust. Students may have come to see that Mark had their best interest in mind, so they followed what he "wanted" them to do. A question remains about what happens when some kinds of authority are foregrounded over others. For example, when personal authority is most pervasive, mathematical justification or rationale might be backgrounded. What might be the impact of such authority structures? These kinds of questions can complement the literature that considers possibilities for teachers to shift authority structures (e.g., Hufferd-Ackles, Fuson, and Sherin, 2004; Skovsmose, 2001). We wonder also how much of Mark's perceived need to establish authority in his new context was necessary and whether his perceived need might change over time. For example, if we had followed Mark through the first few years in this new context, might we have been able to see these categories shift and change as Mark negotiated authority with his students? It is important to be an authority in mathematics and to be in authority to some extent as a teacher, but it is also important to establish a routine in which each student sees him/herself as in authority of his/her own learning so that s/he too could become an authority in mathematics. The students themselves probably had similar struggles—wanting to be independent of Mark while depending on him for guidance in various ways. Yet, little is known about the tension.

Third, Mark's reflections on his authority and attempt to have explicit discussions with students about authority highlighted for us the importance of collaborating with teachers when we explore authority. We note Mark's explicit discussion about authority with his students, which would not have happened prior to our work together on authority. The importance of mathematics teachers "stepping out" or having meta-conversations about norms has been demonstrated (Cobb, Yackel, & Wood, 2003; Rittenhouse, 1998), but we have not seen such attention in the literature about other important aspects of norms. Authority is central to these norms, so we argue that meta-conversations about authority in mathematics classrooms can help students come to terms with their mathematics. Further research on teachers using such strategies is needed.

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