MATHEMATICS TEACHERS' REPRESENTATIONS OF AUTHORITY

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Abstract:

Issues of authority abound in education and schooling but have not been interrogated sufficiently. We describe a tool that we have developed to initiate dialogue with teachers about authority in their classrooms—using a diagram to represent authority in their classrooms. Our analysis of the diagrams mathematics teachers created and discussed in our work with them illustrates the importance of understanding teachers' perspectives about authority. To understand better how mathematics teachers think about the authority in their classrooms, we investigated what sources of authority they represented in their diagrams, and how the teachers related these sources to each other. The diversity in the teachers' representations exceeded our anticipations, indicating that research on authority in classrooms has merely scratched the surface of understanding the ways mathematics teachers think about authority in their classrooms.

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Practices in education and schooling are rife with issues of authority. For example, at the broad system level, authority appears in the monitoring of students' performance on standardized tests and at the level of the classroom, authority occurs in the teacher-student relationship. Yet, in much of the research on teaching and teacher education, authority issues are not treated as a central object of inquiry. Rather, authority sometimes appears as an object of attention in the literature review but with little discussion in the findings, as it did in Brown's (2009) exploration of teaching for social justice. More often authority appears in the findings with no discussion, as it does in the many articles in this journal (and others) that have one or two instances of the word. In a few articles, authority has appeared more substantively. De Freitas and Zolkower (2009), for example, suggest questions about authority for leading professional development, but they do not report on the use of such questions and thus do not show us how teachers think about authority. These theorists instead tell us a little about their own views on authority and how it is manifested in classroom discourse. Other scholars interpret classroom data in terms of authority relationships without attending to what classroom participants say or think about authority. Amit & Fried (2005) provide the most substantive work on authority in mathematics education contexts, with theorization that is substantiated with some interviews with students in order to better understand their perspectives.

These kinds of studies contribute to our understanding of authority in schooling but do not provide us with much insight into how mathematics teachers think about authority. Yet, we would argue that knowing how mathematics teachers think about authority is imperative to understanding issues of authority and agency in mathematics classrooms because teachers have been granted institutional authority to make decisions about what does or does not happen in their classrooms. We have come to make authority issues central in our work because it has been a looming concern of many of the secondary teachers with whom we have worked (e.g., Herbel-Eisenmann, Drake, & Cirillo, 2009).

We see mathematics education as a paradigm context for studying issues of authority in education. Questions about authority are central in mathematics and mathematics education because of the discipline's characteristic interest in truth and proof. Mathematical proofs are supposed to be true regardless of the status or authority of the people involved, thus mathematics is valued for its role in democracy. Ernest (2009), for example, pointed to the sense that mathematical logic can trump authority: "In principle, mathematics is a highly democratic rational discipline in which knowledge is accepted or rejected on the basis of logic, not authority" (p. 59). But how are truth and value established in mathematics? And there are broader questions about authority in mathematics teaching. For example, who should decide what mathematical questions or issues are worth pursuing? On what basis should these decisions be made?

In a recent computer-aided quantitative investigation of a large body of transcripts from secondary mathematics classes, Herbel-Eisenmann, Wagner and Cortes (2010) corroborated the prominence of authority in mathematics classroom discourse. This analysis showed that "stance bundles," which indicate authority relationships, were significantly more prevalent in mathematics classroom discourse as compared to other contexts in which similar analyses had been done. The pervasiveness of authority issues in the discourse may seem to suggest that classrooms focus on questions about proof and truth as suggested above. This study, however, showed that authority structures encoded in mundane phrases in the classroom were commonly contingent on social positioning rather than proof-related discourse. For example, the most common word combination was "I want you to," which suggests students do things because a person in authority told them to, not because it is a good choice or a logical necessity. Authority was unquestioned and placed in the teacher and in accepted mathematical procedures instead of being a result of justified statements. An important and related point from this research is that the teacher-researchers involved in this work were also involved in offering their interpretations of these mundane phrases. In many cases, their interpretations were similar to those offered by the university researchers. In some cases, the teacher-researchers were much more critical of these authority structures than the university researchers.

We see this kind of engagement with secondary mathematics teachers in reflection and dialogue about authority in their classrooms as a necessary contribution to research on authority in mathematics classrooms. These interactions may be part of research on teachers' views on the authority structures in their classrooms and/or part of professional development. In the same way that many mathematics education scholars would argue that understanding how students think about rational number or problem solving can help to improve the teaching of rational number or problem solving, we argue that understanding how teachers think about authority must be the basis of teacher educators' work with teachers on issues of authority.

In this article we describe a tool that we have developed to initiate dialogue with mathematics teachers about authority in their classrooms—using a diagram to represent how they

think authority works in their classrooms--because we have found that it is important to start from their perspectives in this work. We analyze the diagrams teachers created and discussed in our work together. In doing so, we illustrate the importance of understanding teachers' perspectives about authority. These diagrams help answer our research question: how do mathematics teachers tend to think about authority in their classrooms? Our research explores a way of drawing out teachers' thinking about authority in their classrooms and our analysis of their diagrams gives us insight into how teachers describe it.

After outlining relevant literature on authority and articulating our theoretical framework for examining authority, we will describe the three research contexts we have worked in to delineate the sources of the data for this study, including teachers' authority diagrams and transcripts of the discussion about these diagrams. We then demonstrate the diversity of teachers' views on authority and show how their views are contextually grounded. In the final section of the paper, we consider what we have learned from the data and discuss some complexities associated with addressing authority in professional development.

1 Authority in Classrooms

Authority is one of many resources teachers employ for control and our starting point draws on educational researchers who define it as "a social relationship in which some people are granted the legitimacy to lead and others agree to follow" (Pace & Hemmings, 2007, p. 6). This relationship is highly negotiable. Students rely on a web of authority relations with friends and family members as well as with the teacher (Amit & Fried, 2005). Although Pace and Hemmings and Amit and Fried consider authority to being negotiated between *people*, we think tools and artifacts can also be considered authorities, especially in the context of schooling (and we will show that some mathematics teachers share this view). Therefore, we extend Pace and Hemming's definition of authority to include relationships between people as well as relationships between people and tools/artifacts they use. Mathematics textbooks, for example, are often used as a source of authority on answers: mathematics students regularly check to see whether their answers are correct by looking in the back of their mathematics textbook. We recognize that this kind of "checking" is probably more pervasive in mathematics than in literacy or other content areas because a correct answer can be provided and is often the goal of mathematics teaching (e.g., Smith, 1996). Thus, the role of a textbook as an authoritative artifact may be more enhanced than in other subject areas. When a student is asked to interpret a poem, for instance, an answer would be difficult to provide in the back of a book and often there is more than one correct way to interpret the text. Thus, in this review of literature, we consider research on authority between/among people as well as authority ascribed to tools or artifacts.

1.1 Teacher Authority

Educational research related to teacher authority often makes distinctions between different types of teacher authority (e.g. Amit & Fried, 2005; Pace & Hemmings, 2007). Most relevant are the distinctions made between teachers being an authority because of their content knowledge and being in authority because of their social position (e.g., Skemp, 1979) – teachers are "an authority [of content] in authority [by virtue of position]" (Russell, 1983, p. 30). Many scholars argue that the former is more relevant to teachers because it emphasizes their ability to reach their educational goals. Although these distinctions are made for analytic purposes, Pace (2003) has shown that the types of authority become blended as participants interact in classrooms. This blending is also demonstrated in Herbel-Eisenmann, Wagner and Cortes' (2010) corpus analysis.

Skemp (1979) noted that when authority is gained by position, authority is imposed: the teacher commands, students obey, and instructions are perceived as orders. In contrast, authority by knowledge involves being more like a "mentor." The authority is vested by virtue of the person's own knowledge; instruction is sought and is perceived as advice. Amit and Fried (2005) point out "one turns to an expert authority for instructions, not, by contrast, for a discussion" (p. 148). Rival and conflicting values complicate authority relations because they are socially constructed in the service of a moral order (Pace & Hemmings, 2006). Moral order, in this case, was defined as "shared norms, values, and purposes" (p. 21). As Dornbush and Scott (1975) argued, relations of power and control are, in fact, justified by rules and social norms. Teachers and students can share authority for these rules and norms, which then also comes to permeate sharing authority for the mathematical content because the flow of information is also being mutually determined through turn-taking and contributions (Amit & Fried, 2005). As Boaler (2003) argued, as a teacher's authority becomes 'weakened' through sharing it with students, a teacher can deflect her authority to the discipline itself and its logical, rational structure. When authority relations are not clear, however, conflict and power struggles can emerge.

Regardless of what kind of authority seems to be at play, Wilson and Lloyd (2000) contend that teachers need to develop an internal sense of authority, or a sense of agency, rather than rely on external forces in order to develop their own "pedagogical authority." A challenge these authors identify is the fact that sometimes teachers' conceptions of mathematics make it difficult to share authority. Sharing authority can require deep understanding of mathematics that allows a teacher to understand how students are making sense of the mathematics, to draw on multiple solutions and representations, and so on. It also requires balancing various classroom structures (e.g., movement between whole class and small group) and maintaining some kind of semblance of order. Whether teachers feel comfortable doing all of these different things can influence when, how, and why they enact practices that share authority with students.

Alongside the prompt for teachers to develop pedagogical authority, Wilson and Lloyd (2000) also made a parallel argument for how teachers help students develop their own sense of mathematical authority. That is, the same kind of reliance on internal authority can help students learn mathematics with meaning. As Schoenfeld (1992) pointed out, however, the development of internal authority is rare in students, who have "little idea, much less confidence, that they can serve as arbiters of mathematical correctness, either individually or collectively" (p. 62). In fact, students may rely on the consensus of peers and the teacher before they consider their own authority in making decisions about the correctness of their work (Tsui & Ng, 2010).

Up to this point in the article, we have discussed authority relationships between teachers and students and mentioned that family and peers can serve as authorities in deciding on whether answers are correct or not, which is the scope of the actors identified in Amit and Fried's (2005) theorization of authority. In classrooms, however, other pervasive presences that influence authority relations are tools that are made available. For example, textbooks play a prominent role in what and how content is taught, especially in mathematics and science classrooms (Alexander & Kulikowich, 1994; Begle, 1979; Tobin, 1987; Usiskin, 1985). Encouraging students not to be overly reliant on textbooks as their main authority can be a challenge (Wilson & Lloyd, 2000). Additionally, other authoritative texts like dictionaries (see Tsui & Ng, 2010) or tools (like graphing calculators) have been shown to be resources students rely on as they consider the correctness of their work. In the following section, we consider some literature related to these other potential sources of authority.

1.2 Other Sources of Authority

Most research on authority in classrooms has focused on teacher authority, briefly mentioning that the textbook may have played a role in authority relationships in classrooms (Haggarty & Pepin, 2002; Amit & Fried, 2005). As Olson (1989) argued, the separation of the author from the text as well as the particular linguistic characteristics of a textbook help to instantiate the textbook as an authority. Textbooks, thus, constitute a distinctive linguistic register involving a particular form of language (archival written prose), a particular social situation (schools) and social relations (author-reader) and a particular form of linguistic interaction (p. 241). Yet, as Baker and Freebody (1989) contended, the authority of the textbook in practice is the result of how the textbook is used in the classroom. Their perspective took as central actual classroom interactions and the authors empirically investigated how "textauthorizing practices...may be observed in the course of classroom instruction" (p. 264), as well as how these practices evolve in relation to the authority of the teachers. To illustrate these practices, the authors examined the kinds of questions teachers ask and the ways teachers responded to students' answers to their questions. They sought to "describe the intimate connections between talk around text and the social organization of authority relations between teachers and students. Teachers may be shown to use various practices to assign authority to the text and simultaneously to themselves" (p. 266). For example, Herbel-Eisenmann (2009) illustrated how a teacher might end an interaction by asking students to read from the book in order to help students produce the "correct" response to what appeared to be an open-ended question at the beginning of the interaction. By doing so, the ideas and words no longer originate with students. The textbook's voice is privileged instead.

Other resources or tools that may be privileged as an authority uniquely in mathematics classrooms include things like calculators or graphing calculators or mathematics manipulatives (Ball, 1992) like base ten blocks. For example, Glasgow and Reys (1998) found that when they gave students incorrect answers on a graphing calculator, students were resistant to questioning its accuracy. Although other researchers (e.g., Williams (1993); Wilson and Krapfl (1994)) have raised similar concerns, Doerr and Zangor (2000) did not see the graphing calculator become an authority in their research in a secondary mathematics classroom. They attributed this finding to "the teacher's knowledge of the limitations of the calculator and her belief that conjectures are proven on the basis of mathematical reasoning or argument" (p. 159). This finding speaks to the influence a teacher might have in helping students develop the kind of agency for their own learning that others have suggested. Teachers play a role in mediating the authority of resources like graphing calculators, just as Herbel-Eisenmann (2009) and Wilson and Lloyd (2000) showed they played this role with textbooks.

2 Theoretical Lens for Making Sense of Authority

We see the idea of positioning as being important to this work because it recognizes that relationships necessarily involve issues of control, authority, and power. These issues appear at many levels, including interactions within a classroom (in one-on-one interaction, small groups

and whole-class interaction) and between people in the class and stakeholders outside of it. We can identify issues associated with positioning by looking at turn-by-turn interactions in student group work or in whole-class discussion. The issues can also be identified by studying documents and policies that reflect and structure relationships among wider stakeholders, including, for example, curriculum documents that specify learning outcomes, standards documents published by the National Council of Teachers of Mathematics (which inform curriculum documents in Canada), provincial policy documents that stipulate common assessment regimes and the way these assessments are used, and local school authority policies. Children are positioned in certain ways by the texts from each of these sources just as they are positioned in human interactions in the classroom.

Harré and van Langenhove (1999) describe positioning as the ways in which people use action and speech to arrange social structures. 'Positioning' can refer to physical positioning, as it does in Goodwin's (2007) study of how parents and children physically position themselves while the children do mathematics homework, but more often 'positioning' is used metaphorically to represent relationships. Harré and van Langenhove's description of positioning carefully addresses the role of positioning in interaction through a focus on "storylines."

As outlined by van Langenhove and Harré (1999) clues in word choice or associated actions evoke images of known storylines and positions within that story. For example, a teacher may say something that positions herself as a coach and the student as a motivated athlete. The student may continue the interaction complicit with this positioning or resistant to it. Alternatively, the teacher and student might be operating under different assumptions about what storylines may be at play. For example, the teacher may see herself as a coach while the student sees her as a drill sergeant. This multiplicity of possible storylines demonstrates that various authority relationships may be envisioned simultaneously in any particular situation.

Most authors, including ourselves in previous work, who draw on positioning theory or the metaphor of positioning to interpret data represent relationships specifically between people —for example, in transcripts of classroom interactions or transcripts of interviews. We diverge slightly from this pattern by focusing instead on what teachers say about the positioning in their classrooms through their drawn representations and their talk about these representations. Rather than identifying positioning, this article attends to what teachers say about positioning and how they represent the positioning in their classroom relationships, from their perspective as participants in the classroom discourse.

This distinction between the actual positioning or authority structures and what people say about these power relations is significant. Using interviews with mathematics students, Amit and Fried (2005) claimed to look at the way authority actually is in classrooms, but one might argue that they were only looking at what students *said* about authority. However, authority is only a conceptualization, so like with positioning theory, there is no empirical authority relationship. There are only people's perceptions or attributions of authority. Nevertheless, Amit and Fried noted differences between what students did and what they said about their choices relating to authority.

Positioning theory and Amit and Fried's theorization of authority share the characteristic that they are not empirical but rather conceptual. Thus any account of authority is contestable. Related to this, both theorizations note the necessity of complicity for a stable relationship of authority or positioning. Just as positioning requires both parties to share a similar sense of the storyline at play, one only has authority when others consent to it, according to Amit and Fried.

3 Method

The primary data in this article are the diagrams generated by teachers to describe the way they see authority at work in their classrooms. Drawings have been used as research tools and have been shown to provide opportunities for prospective and practicing teachers to explore aspects of identity (Black & Halliwell, 2000; Katz et al, 2011). Researchers have argued, in fact, that drawings can bring forth often unarticulated and deeper meanings than verbal contributions might offer.

The diagrams in our research were supplemented by transcripts of our recorded dialogue with teachers, as they described their diagrams and asked questions of each other during such dialogue. The three contexts for these dialogues varied, though they are all set in research studies that were oriented primarily around professional development for participant teachers, and they were all set in Eastern Canada. Because of the professional development focus of each research context, we focused on prompting reflection and discussion among participants. The diagrams and recordings (and transcripts) of discussions are also useful for addressing important questions for research. Our fundamental question is "How do mathematics teachers think about the authority in their classrooms?" We focused this question for this research by asking: 1) What sources of authority do teachers represent in diagrams depicting classroom authority? 2) How do teachers relate these sources to each other and to the contexts in which they work?

3.1 First Context

The first context in which we had teachers draw diagrams was part of our study engaging three secondary mathematics teachers, Jill, Mark, and Dawn, in conversation about authority structures in their classrooms (All names of teachers are pseudonyms). We met bi-monthly with the teachers for three years, and periodically recorded (video and audio) and transcribed series of lessons from each of their classroom teaching contexts to discuss in the meetings. The meetings were also recorded and transcribed. The goal of the research and the conversations was to explore practices that can help mathematics teachers to develop their repertoires for addressing issues relating to the role of authority in their teaching

At the outset of this research, we interviewed each teacher independently and asked him/her to describe his/her view of authority in his/her classroom. Immediately prior to giving instructions for drawing their diagrams we asked the following questions about authority: 1) What or whom do your students see as authorities in your classroom? 2) How do your students know something is right in mathematics? 3) How do your students know what to do in mathematics? 4) How do you, as a teacher, know what is right and what to do in mathematics? After listening to the teachers' answers to these questions, we drew for each teacher a thick dot on a blank paper or blackboard and said, "This dot is you." We then invited them to use symbols, lines, words, or whatever they needed to show how authority works in their classroom to complete the diagram. While the teachers were drawing, they talked about what they were drawing, and we asked them to explain some of their choices. For example, we asked questions like, "What does this arrow mean?" or "Why did you use a dotted line to connect those parts instead of something else?" The diversity in their diagrams and descriptions fascinated us, prompting us to use this line of questioning in our other interactions with teachers and preservice teachers, both in and outside of research contexts.

3.2 Second Context

The second research context from which we use diagrams for this article was a two-day, seven-session professional development for twenty-one grades 6 to 9 mathematics teachers in a city in Eastern Canada. The session focused on discourse in mathematics classrooms and was led by six mathematics education researchers. The authority diagram activity was the first session for half of the participant teachers and the fourth of seven sessions for the other half. All the sessions were video-recorded and transcribed. In this context the activity was different from our first context because in our first context the teachers were being interviewed as individuals. In this second context the teachers were at tables. They did not talk about their answers to the four questions about authority. Instead they were given time to make some notes and reflect on those questions before the diagram. When drawing their diagrams, they could probably see some of their colleagues' diagrams in their periphery and they did not talk during that time. When the diagrams were complete, they in turn described their diagrams to the group. For each description, the group was invited to ask questions and make comments. In particular, questions of clarification were encouraged - "Why did you ... in your diagram?" For this discussion the group moved from table to table. The teachers at a table described their diagrams while the others stood in a circle around the table.

3.3 Third Context

The third research context from which we take diagrams for this article was another twoday session in a different city in Eastern Canada, partly modeled on the two-day session described above. The ten grades 6 to 8 mathematics teachers' first activity was drawing authority diagrams as done in the second context. There was a difference, however, in the format of the discussion of the diagrams. The teachers came to the front of the room to describe their diagrams, which were placed on a document projector. Questions and comments were encouraged, as in the second context. As with the other contexts, the entire session was videorecorded and transcribed.

3.4 Articulating the Specificities of Our Diagram Instructions

Some of the differences among the diagram contexts related to unavoidable particularities of the settings. There were choices we had to make, however, about how to set up the diagrams – for example, we had choices about how to direct the teachers to draw their diagrams. We have been asked why we asked teachers to represent themselves with a dot. We could have left it open as to how or where they represented themselves in their diagrams. In our view, using a non-dimensional dot to represent the self helps to focus the diagrams on the relationships and interaction more than on personal identity. If a teacher were to think about how she would represent herself, as was the case for teachers in other studies we found involving teachers drawing themselves (Black & Halliwell, 2000; Katz et al, 2011; Weber & Mitchell, 1996), we would expect the focus of her attention in the exercise to be significantly different. With a non-dimensional dot, her representations of herself would more likely be in terms of her relationships with others, rather than on identity markers (e.g. gender, clothing).

Our rationale for asking teachers to create authority diagrams at the outset of our interactions with them included a number of considerations. We were interested to see how they

thought about authority before we started talking with them about authority; their diagrams and what they revealed about the teachers' views were important to our developing conversations. This is in line with the findings of Black & Halliwell (2000), who showed that drawing helps teachers examine reflectively the connections between their feelings, aspirations, past experiences, and relationships.

Furthermore, starting the conversations by listening to them would demonstrate our recognition of the teachers as professionals who have experiences to share in the ongoing conversation. Listening to them would position them with authority, we hoped. Unfortunately, we have no way of seeing whether this intended effect was successful. In the ongoing conversations, there were examples of the teachers demonstrating their authority but there were also examples of them appearing to be relatively passive. We cannot compare these conversations with the conversations that would have taken place without doing the diagrams. We have evidence, however, that the diagrams continued to be important to the teachers; the group of teachers with whom our interaction was the longest (context 1) continued to refer back to their diagrams throughout our work together. For example, at a mathematics teacher conference presentation more than two years after the teachers drew their authority diagrams, Mark and Jill showed their diagrams and used them to reflect on their growth as teachers. Mark showed his original diagram with added modifications in another color.

Though there were differences in the contexts in which the teachers drew their authority diagrams, even within any one of the contexts there were further differences among the teachers' background experiences and their aspirations. Thus the differences in the contexts ought not to be a significant factor in the analysis of the diagrams.

3.5 Analysis

We analyze all 34 teacher-diagrams produced in these three contexts: 3 from individual interviews in the first context, 21 from the second context and 10 from the third context. Our analysis of the diagrams was responsive to themes and trends that we observed in the diagrams and what teachers said about their diagrams. We had not planned in advance how we would analyze the diagrams. We began the analysis with an organic identification and categorization of depicted 'sources' of authority. We found that it was not straightforward to decide what in a diagram represented a source of authority. Whether or not teachers who identified these 'sources' talked about them as showing authority, we call them 'sources' because they are *potential* sources. Furthermore, even a passive person might be called a source of authority because his or her acquiescence to another person in a relationship is part of what gives that other person her or his authority. Acquiescence makes authority possible, and is thus a 'source' of authority. Items we do not take as sources of authority include the arrows and lines, which we took as representing connections between the sources of authority in the diagrams, though we recognize that this exclusion is a little problematic. Some of the symbols were clearer than others, but our interpretation of what the symbols meant is informed by the teachers' accompanying discussion about their diagrams recorded in transcripts.

We decided to organize the sources into groups like people, bodies of people, disciplines and physical materials as sources of authority (or at least potential sources). For some depictions we depended upon our recordings and transcripts of the teachers talking about their drawings to identify the objects. Whether or not we needed this commentary for recognition, their commentary helped us understand how they were thinking about these objects. Connecting this commentary with the diagrams also helped us recognize the extensive choices teachers made when drawing, all of which relate to the teachers' conceptions of the sources of authority.

After looking at the objects in the diagrams we focused our attention on the symbols used to indicate relationships among the objects. We individually identified interpretations for these symbols and then met to talk about and formulate a collectively generated interpretation for the relationships that we thought were being expressed. For example, many teachers used arrows to connect objects. The placement and direction of such arrows suggested a teacher's sense of which objects connected and of how they connected. As we discussed our individual interpretations, we collectively came to see these symbols connecting objects as being a form of metaphor. This is in line with Zwicky (2003) who identified metaphors as forms of seeing-as: "Metaphor is a species of understanding, a form of seeing-as: it has, we might say, flex. We see, simultaneously, similarities and dissimilarities" (p. 4). Because the teachers were drawing, an action that is by nature spatial, their representations of their understanding of authority in their classrooms were spatial metaphors for diverse experiences, including love, for example. Some of the symbols used to indicate sources of authority were metaphorical too, but not necessarily spatial metaphors.

It was not always clear to us from the diagrams themselves, which symbols were the primary ideas for the teachers doing the drawing. For the teachers whom we watched drawing, it was more clear which symbols were primary because we saw what they drew first. It appeared to us that most teachers drew their sources of authority first and then connected them, but we believe some of the teachers began their drawing with the metaphorical symbols and added in sources afterwards. Nevertheless, our interactions with the teachers facilitated our understanding of what in their diagrams were of primary importance to them and what their symbols meant to them.

Though we refer to our interpretation of the diagrams as being subsequent to the interactions with the teachers, we recognize that interpretation began with our first access to the diagrams, even while interacting with the teachers. Our questions of the teachers during these interactions required our interpretation. Finally, for the teachers in the first context, we were able to consider their diagrams in the context of three years of ongoing interaction that involved group meetings, individual interviews, recordings of them teaching and interviews with their students.

4 Teacher's Views on Authority: Findings and Discussion

Our description of the authority diagrams follows our progressive interpretation of the data. This development is reflected in the two questions that focused our fundamental research question. First, we discuss the variation across the diagrams. We take items in the diagrams as indicators of sources of authority identified by teachers (which addresses our first subquestion) and the connecting symbols in the diagrams as indicators of the kinds of metaphors used to think about mathematics classroom authority (which addresses the first part of our second subquestion). Secondly, we illustrate how the diagrams can be seen as unique expressions of individual teachers in their particular contexts by looking more closely at the diagrams and discussions of the three teachers with whom we worked for three years (which addresses the second part of our second subquestion). We do this, in particular, to explore in more depth the relationships between the sources and the ways in which the teachers link their representation of

authority to their particular context. Throughout this section, we interpret the findings, making connections to the literature where relevant.

4.1 Sources of Authority

In this section we explore the variation in the diagrams. We begin with an account of the differences in who and what teachers included in their diagrams, which we refer to as sources of authority. Table 1 lists the items shown in the 34 diagrams with their frequencies of occurrence in the diagrams. For example, in the first column we have "self (34)", which means that 34 of the diagrams included a depiction of the teacher's self. Of course, we asked them to start with a dot to represent themselves, so it is not a surprise to see all 34 diagrams including the self. Similarly, in the third column we have "textbooks (10)", which means that 10 of the 34 diagrams included an image or text that represented a textbook or textbooks. The processes/actions in the second column tended to be text from the diagrams – for example, two diagrams included the word "direction" (in the chart we listed all the processes with 'ing' endings though the teachers did not write them all in this way). Most diagrams had aspects of direction at play as they depicted directed communication among people, but only the two diagrams place "direction" as an object. We have considered further subdivisions of each column in the chart but we do not think such divisions would be warranted unless we were to have had more extensive conversations with the

people	processes/actions	classroom objects	disciplinary artifacts
self (34)	questioning (5)	textbooks (10)	math curriculum (10)
students (22)	communicating (3)	black/whiteboard (7)	prior skills (4)
family (7)	discussing (3)	calculator (4)	ideas (3)
other teachers (4)	directing (2)	computer (3)	prior problems (3)
administration (3)	giving feedback (2)	desks (3)	expected answers (2)
department of	answering (1)	manipulatives (3)	classroom rules (1)
education (2)	confirming (1)	books (2)	daily routine (1)
professional	comparing (1)	materials (2)	methods (1)
learning	discovering (1)	posters (2)	prior experiences (2)
community (2)	disrupting (1)	resources (2)	questions (1)
church (1)	estimating (1)	ruler (2)	roles (1)
groups (1)	focusing (1)	handouts (1)	tests/exams (1)
NCTM (1)	guiding (1)	paper (1)	"There is no math god"
school board (1)	instructing (1)	technology (1)	(1)
sport teams (1)	investigating (1)		topic (1)
tutor (1)	justifying (1)		
vague others (1)	memorizing (1)		
	planning (1)		
	positioning (1)		
	practicing (1)		
	prompting (1)		
	raising hand (1)		
	understanding (1)		

 Table 1 items represented in 34 authority diagrams and their frequencies

teachers about the types or categories of authority they represented. We organized the sources into four columns to facilitate discussion of some of our observations here. We suggest that there would be significant potential to investigate these observations further in future research.

We see more diversity of sources in these diagrams than what we have previously seen reported in the literature. For example, we have seen self, students, families, and peers as sources of authority in prior work but have not seen professional teaching organizations appear in literature on authority in classrooms. Additionally, the category we label as "processes/actions" encompass a range of sources that we have rarely seen in work on authority. Because there is such an extensive list in each column, we highlight a few ideas about each of the columns rather than discuss each item teachers included.

As noted above, every diagram included the self, the dot in the center. Our choice to specify that the teacher be represented as a dot likely influenced the teachers' choices of how to represent others in their diagrams – for example, many teachers used dots to represent students and other people as well.

Some teachers used symbols other than dots to represent individuals. The teachers' comments on their diagrams made it clear that some of these symbols were chosen because the teachers wanted to say something about different identities of students, which would not be expressed in the students' interaction with their neighbors. (They could use dots for each person if they wanted to focus on the interactions.) Some of these teachers talked about wanting to indicate that students made choices in their classrooms. In these cases, the different depictions of students might say less about individual student identities and more about the teacher's understanding of the choices a student could make on any particular day.

Even among the diagrams that use dots for people, there are distinctions between kinds of dots. Dawn and Jill (both from context 1), whom we will discuss in greater detail later, used open and closed dots to distinguish between kinds of people (see Figures 1 and 2, respectively). (Jill's diagram was done on a blackboard, so it could not be scanned. Figure 2 is a model of her blackboard drawing.) Dawn (Figure 1) also used an x to represent the student as distinct from teachers who were dots. Mark (context 1), whose diagram will be discussed later (see Figure 5), used sizes of dots to represent the relative weight of authority ascribed to individuals. Not all students were depicted as equal, demonstrating an awareness of complex differences in relationships—there are not only teacher-student relationships but there are many kinds of student-student relationships.

Other teachers distinguished between students in other ways. For example, Dallas (context 2) used rectangles to represent his students (perhaps these are desks with metonymic connections to the students), and showed some as having questions and others as not engaging (Figure 3). Another teacher used a dot to represent a group of students. We wonder what this view of students as collectives instead of as individuals means for his teaching. For example, many researchers who write about equity issues raise concerns about essentializing students. We wonder whether exploring such an implicit choice to represent groups of students collectively may help researchers better understand the ways in which teachers essentialize students.

The differences among teachers' representations of students may say something about their perceptions of students, but we think they also say something about the lens (or theory) used for reflecting on the classroom. For example, positioning theory focuses on interactions among people in a situation whereas others may focus on the identities of individuals. Differentiating among students with different symbols points attention to individuality and to



Figure 1 Dawn's authority diagram (context 1)



Figure 2 Jill's authority diagram (context 1)

how students bring different experiences and ideas into the classroom. Teachers using only dots to represent students focus on the classroom interactions.

Thirteen teachers focused on actions or processes by using symbols and words (which are also symbols) to explicate what happens in the interactions among the sources of authority. These thirteen teachers came up with twenty-three different processes, which were almost all mentioned only once, to describe the nature of the interaction. This diversity indicates to us that one could expect other teachers making authority diagrams would be likely to include more processes yet. Furthermore, this diversity indicates that teachers' views on authority differ significantly. Sometimes teachers sitting close together when drawing seemed to borrow ideas from each other, but the diversity of responses is evidence that the teachers had significantly different points of view and the desire to express these views. The processes identified here, for instance, questioning, communicating and discussing, have specific importance in mathematics classrooms but they are processes that occur in other classrooms as well. We might expect a different set of processes identified by teachers in other disciplines, if they were probed for



Figure 3 Dallas' authority diagram (context 2)

further specificity. We also note that only one process seemed related to proof (i.e., "justification") and was only mentioned once. Again, however, specificity might need to be probed.

The classroom objects identified as sources of authority are more unique to mathematics classrooms. Manipulatives (concrete objects used to help model mathematical ideas) appeared in a number of diagrams as sources of authority and garnered some discussion when teachers presented their diagrams to each other. When asked to say more about manipulatives as an authority in her diagram, Rochelle (context 2) said,

We're doing algebra now so they need the tiles on the desk with them. If they're not sure if they got it right they can go to their tiles and they can use them to play around, and make sure it [the answer] works. And if your tiles don't give you the same answer, well, that is giving you feedback—to see, to see if they have the right answer.

Manipulatives are inanimate objects, but Rochelle attributed authority to them, much like looking up answers in the back of a textbook or checking one's work on a calculator. Students can "go to the manipulatives" to test their ideas. We find the shift from "they" as the subject in the second sentence to "your tiles" as the subject in the third sentence interesting: the second sentence suggested that students use and act on tiles, whereas the third sentence made the tiles the subject and puts students in the position of direct object (the tiles "give you" the answer). The manipulatives, however, are always controlled by the student and are used as a tool. It is interesting to us that Rochelle and two other teachers (including Louise, see Figure 4 in the next section) represent manipulatives as separate from the self though they are extensions of the self. (Louise, like Rochelle, was in context 2, but they were in different cohorts and thus not in discussion with each other.) This splitting of the self seemed to be similar with the depiction of an idea or calculator in the authority diagrams.

This split is reminiscent of the split self, which is articulated by Rotman (2008). He described how people doing mathematics embody different roles within themselves; the 'thinker' instructs the 'scribbler' what actions to do to inform further thinking. A student's work with manipulatives or with a calculator is an example of the scribbling (or rote action) performed to explore one's thoughts, which seem to operate separately in a conversation within oneself. Similarly, the 'ideas' (in our "disciplinary artifacts" column), depicted by three teachers, may reflect the 'thinker', the other side of the self, described by Rotman.

Of the disciplinary artifacts teachers referenced the mathematics curriculum was the most prevalent source mentioned. This is not surprising to us because the literature includes some attention to the role of curriculum in teachers' decisions. We should note that the word 'curriculum' in Canadian contexts refers to government-published standards documents; textbooks are seen as distinct from curriculum. Nevertheless textbooks were the most prevalently referenced source of authority among classroom objects, just as curriculum was the most prevalent of the disciplinary objects. As we described in our literature review, this prevalence of teachers' references to curriculum/textbooks shows that mathematics education researchers' attention to curriculum/textbooks is warranted. We recognize that textbooks could be in Column 4 in Table 1 because they are artifacts representing the discipline. However, we placed textbooks in Column 3 with "classroom objects" because, unlike curriculum guides, they are objects used by students in the classroom.

We note that there are a variety of textual resources that play out in authority relationships similar to textbooks/curriculum. For example, the blackboard or a PowerPoint® can act as a pseudo-textbook because they can be used in the same way as a textbook. Teachers can refer to them (or even defer to them). They are a way for teachers to place text outside of themselves, and they are media in which teachers choose what to represent or foreground.

The suggestion that "prior" skills, problems, and expectations might be sources of authority was interesting because these items suggest attention to what has previously been described in the literature as "common knowledge" (Edwards and Mercer, 1987) in classrooms. Other disciplinary artifacts appeared as part of the set of the social norms that guided the classroom work such as rules, routines, and roles.

4.2 Positioning of Authority Sources

In addition to the variation among the diagrams in depictions of sources of authority, we found variation in how the sources were arranged in the diagrams. For example, 7 of the 34 teachers arranged their diagrams to depict the physical arrangements of their classrooms. For instance, the diagram done by Dallas (Figure 3) showed the arrangement of desks with himself and the blackboard in front. These diagrams had relatively scaled depictions of certain aspects of the physical arrangements with some figurative additions.

For instance, Louise (Figure 4), one of the three teachers who depicted manipulatives as sources of authority, showed how the desks were arranged in groups in her classroom and showed one wall with some postings on it and a computer center up against it. She positioned herself in the center of the class. The arrows, however, were relatively figurative. She talked about herself at the center moving outwards: "the teacher is in the center and circulates to as many students as possible." (The direction of the arrows must be figurative because it is not possible for her body to move outwards all the time.) She also said that students made choices about how and where to work on their problems thus there were different kinds of arrangements



-teacher visits all students

Figure 4 Louise's authority diagram (context 2)

of her students, but these differences depicted structures of their interaction (or social positioning) more than physical positioning.

By contrast, Mark (Figure 5) did not use arrows in his diagram. He did, however, use lines, which he said indicated his movement throughout the class. He talked about balance without mentioning direction (no beginning and no end): "I'm really all over the place." Mark was the only teacher to talk about the significance of his spatial arrangement in terms of authority, but others did say that the arrangements of desks in their classrooms (not in their diagrams) has a significant impact on the way authority works. Mark and other teachers had their dot at the front of the room in their diagrams, and talked about their position at the front of the room. They did not say that this was significant in terms of how authority works in mathematics, though we would suggest that this physical positioning, who stands in front of the others, has a powerful impact on social positioning in classrooms.

Most significantly, 27 of the 34 teachers did not organize their diagrams to reflect the arrangement of their classrooms. They saw their diagrams more metaphorically, though there were metaphorical aspects even in the seven diagrams depicting the physical arrangements of their classrooms. In most diagrams, the teachers used lines, arrows and other symbols to help represent the metaphoric relationships among the objects in their diagrams.

The metaphors used to connect the sources of authority were as important as the sources themselves. Of the 34 authority diagrams, 24 had arrows connecting people to other sources of authority but there were differences among the arrows. For example, Dawn (Figure 1) said her arrows represented someone *looking to* an authority, and Jill (Figure 2) used arrows to indicate the *flow* of authority in communications, thus Dawn's arrows and Jill's appeared to be in opposite directions. Also, Louise (Figure 4) used broken arrows, which she did not explain in



Figure 5 Mark's authority diagram (context 1)

discussion. These differences in direction and form of the arrows suggested different ways of thinking about authority. Jill's focus on communication may indicate more attention to the negotiation of authority, and Dawn's focus on "looking to" may foreground the expectations that underpin communication, but we can only speculate because Jill and Dawn did not talk about the difference except to clarify their confusion in understanding each other's arrows.

Other metaphors appeared as well. Dallas (Figure 3) depicted a ladder and described a relationship that had him making decisions about what students should do. With this description, Dallas garnered sympathy from at least one other teacher who helped him finish his sentences:

Wagner: What's the railway track?

- Dallas: Oh that's actually a ladder. I have some kids that don't get math. They just shut down when I ask them a question, but if I can say, "Okay, but you know this because we talked about it the other day. You know this, you know that." So then we go up another step and they do good. But it's me trying to break it up evenly. So that's the strategy.
- Woman: That's how they know what to do. You lead them through it.
- Dallas: Yeah, with each individual one it's-
- Woman: It's exhausting.
 - Dallas: Yeah, that's the word.

We note the fact that Dallas highlighted prior knowledge that they "talked about...the other day" as the information that might get them to another step and not other potential sources like logic or conceptual connections to other ideas. The steps become a way to think about "break[ing] it up evenly," which make us wonder if stated procedures are the authority underlying Dallas' ladder metaphor.



Figure 6 Jean's authority diagram (context 2)

Jean's (context 2) diagram employed another metaphor (Figure 6). The mirror and window may not look very central to the diagram, but they were the first aspects of the diagram about which she talked. Before describing the images that represented what she called 'influences' on students, she said, "I thought it was important that kids have a window to see forward and also a mirror to see a reflection of themselves, and that it was important to reflect." She did not say what artifacts students might use as prompts for reflection. Perhaps they used their manipulatives, as described by other teachers, or perhaps they relied on memories. Nevertheless, her comments reminded us of Skovsmose's (2005) suggestion that both background and foreground are important for working with students—background refers to their prior experiences and foreground to their aspirations. Jean's drawing and comments also reminded us of Gutiérrez (2011), who also used a mirror/window metaphor to say that students should be able to use mathematics to look out at the world but should also be able to see and recognize themselves in the mathematics. If we had had the opportunity, we may have probed further for potential identity development and equity concerns underlying this metaphor.

Yet another metaphor was described by Joanne (context 2, Figure 7). She saw authority as something that can be passed from one person to another but in the short time given to think about this she was still seeking imagery that recognized the fluidity of relationships and authority.



Figure 7 Joanne's authority diagram (context 2)

If I'm in the center, I drew a spiral, meaning that I'm everywhere in the classroom. I looked at it as communication. And authority is kind of passed on to the students through giving them, I guess, the understanding or the ability to problem solve and to work through and to help each other. So it leaves me anywhere along here you can find students. And there's arrows going every which way so it's been passed between the students and between myself. And I talked about the ripple effect. Even though the authority might be given to me to teach these students then ultimately I'm passing it on to them to conduct their own learning. So it's more of an economist's effect. I drew those [radial lines] kind of like a radar, you know. It's kind of pulsing out, [it's kind of like a] spider web, interconnected. I was going to have the web and the radar and the ripple effect all happening at once. The important thing is I'm not anywhere per se and the students aren't in any exact position. It's all transient.

Oyler (1996) challenged possession metaphors for authority. One problem with such metaphors is that they suggest authority is like a finite resource and that one person's increased authority implies someone else's loss. Joanne's description demonstrates for us her struggle to conceptualize authority differently from the dominant possession metaphor but she still used some possession metaphorical language.

4.3 Diagrams as Representations of Teachers' Particularities and Contexts

In order to show the connections among teachers' authority diagrams, their particular contexts and points of view, we discuss in greater depth the diagrams from the three teachers who did their diagrams in interviews (context 1). We chose these three because we have the most insight into their contexts, having worked with them over three years. We have visited and often recorded their classrooms, interviewed their students, and met with them as individuals and a group over this period of time as part of a research project aimed at developing effective ways of engaging teachers in dialogue about authority in their mathematics classrooms. The exercise of drawing the authority diagram, for the three teachers, contributed powerfully to our developing understanding of them as teacher educators who were collaborating with them. Each teacher who drew a diagram for us in other situations would have an equally rich context that informed their diagram choices, but we did not have as much interaction with them to learn about their contexts.

Dawn drew icons and other symbols representing the different sources of authority in her classroom (Figure 1) around the black dot representing her. From her talking while drawing, we know that an x represents a student, another black dot represents other mathematics teachers, an open dot a tutor, and other symbols represent textbooks, rulers and calculators. As she introduced each source of authority, she drew arrows to show where one looks for authority. For example, the arrow from a student to Dawn indicated that the student looked to her as an authority. When showing her diagram to other teachers later, Dawn noted other sources of authority as well. Her diagram represented some of the relationships, demonstrating that there were many authorities at play.

Dawn's conception of authority in her typical classroom is reminiscent of Amit and Fried's (2005) web of authority relations as she noticed a variety of sources of authority. Dawn, however, also drew attention to inanimate objects as authorities, which were not part of Amit & Fried's theorization—calculators and textbooks, for example. After drawing her diagram, Dawn reflected on it and was struck by the various objects she included in it as authorities: "So I guess these would just be tools. They could be any kind of tools that students would be using." We note that even inanimate objects, such as textbooks, can be considered within human relationship by drawing attention to author (for textbooks) and designer (for calculators) choices. Authorship, what Povey (1997) playfully referred to as author/ity, is an important part of authority structures. Dawn also drew more attention than Amit and Fried to people related to the academic institutions, namely other teachers and tutors, but left family members out. In fact, she considered the possibility and rejected it: "You don't get the kids saying 'my parents told me' as much."

Jill completed the diagram (Figure 2) by drawing empty circles for students around the black dot representing her, and then arrows to show the direction of authority. As noted earlier, her arrows were different than Dawn's. Jill talked about the arrows as showing the direction of the communication of understanding. Her descriptions accompanying the drawing of arrows showed them to be more complex than representations of just any communication. For example, an arrow from her black dot to a student's open dot indicated her showing the student something that she understood or knew, and the student understanding *and accepting* her knowledge. Not all students understand or accept all they hear, thus only some students received arrows. Similarly, some students who did not understand or accept her mathematics managed to find understanding in conversation with other students, and were able to show Jill their knowledge in a way that Jill accepted. This was the way she described the arrows coming back to her: "they're

coming back to me with better questions, and I'm thinking wow, they really took that one further." Not all communications from her students counted as an arrow.

Jill's diagram is reminiscent of diagrams in education literature showing paths of communication, though her conceptualization of the arrows were more sophisticated, in a way, because they represented *accepted* communication, not simply communication. She showed no external influences. She said in the interview that she tried to focus her attention on the students themselves. She listened to them and interacted with them as an individual herself, not as a representative of something beyond the reach of the students When we asked her how she used textbooks, she said "it's having less and less authority every year I use it [...] I'm starting to look more at what did my kids do." For example, she modeled what Schoenfeld (1992) referred to as internal authority as she justified the ideas she wanted to communicate in terms of the experiences and prior knowledge of her students, not by appealing to a book for authorization. This is like Harré and van Langenhove's (1999) positioning theory, which focused analysis only on immanent presences, nothing external.

Mark completed his diagram (Figure 5) with a physical representation of the classroom, showing the arrangements of students, who were smaller dots, the blackboard (the straight line), a bookshelf with texts (also authoritative dots) that students can refer to, and his desk at the back of the room. The curvy lines indicated his movement throughout the room. Some students have larger dots because they, like him, were recognized as having more mathematical authority than the others. When drawing, he talked about balance. Authority should be spread throughout the classroom, he said. Thus he arranged seating plans to spread the students regarded as authorities around the room, and he himself moved around to avoid fixing authority in one place: "I'm really all over the place [...] I try to distribute the authority as much as possible." His aim to distribute authority as much as possible is reminiscent of the broadly distributed authority described by Cobb, Gresalfi & Hodge (2009) based on their interviews with mathematics students talking about their obligations and competencies. However, his elaboration suggests differences from their conceptualization of distribution. He talked about the distribution of authority as spatial. He saw himself covering space, and with his seating arrangements he spatially distributed students seen as authoritative. This aspect of his conceptualization was quite different from anything we have seen in the literature on authority in classrooms, yet we find his elaboration interesting and compelling. It relates to positioning, but unlike most scholarship on positioning that uses physical relationships as metaphors for interpersonal relationships, his conceptualization recognized the effect of physical positioning. We think that physical arrangements are significantly related to human interpretations of relationships in any given situation.

When each of the three teachers described their diagrams to one another, they all found each other's diagrams and explanations informative and true representations of some of their own views on authority. They attributed some of the differences to their different personal experiences and teaching situations. Because Dawn taught mathematics in a French Immersion setting, there were two disciplines that are often seen in competition for priority – mathematics learning and language learning. Thus it did not surprise us that her conceptualization of authority showed awareness of multiple sources of authority. Jill had many Aboriginal students, with a culture that is very sensitive to human relations and that has a long history of tension with external colonial powers. Thus we were not surprised that her conceptualization of authority focused on the human relationships immanent in the classroom and ignored external powers. In addition to being a teacher, Mark was a coach who was running sports camps every summer and advising for teams beyond his normal coaching. His drawing reminded us of play sheets, and he

talked about the need for every student (like every player) to follow the directions of the "coach" at the same time as they make decisions for themselves within the coach's system.

We recognized connections between classroom context and the authority diagrams for Dawn and Jill, and we recognized connections between Mark's personal context and his authority diagram. It is important for us to clarify that these connections were ones we identified; the teachers themselves did not explicitly make these connections. They did, however, take ownership of these connections and continued to talk about their diagrams and the connections we identified between their diagrams and their contexts as we worked together over the years. We expect that there are further connections between these teachers' conceptions of authority and their personal and professional contexts. When we asked them if they saw further connections they did not identify any. This leaves us wondering whether their (lack of) response reflected the difficulty of noticing such connections in one's own experience, the difficulty of introspection, some other difficulties or a combination of these difficulties. Nevertheless, in addition to seeing contexts influencing how they represented authority, we see evidence of the way they think about authority influencing the way they structure their classroom interactions, though this is soft evidence—difficult to articulate (like a gut feeling). We have elaborated on Mark's classroom practices in relation to authority in Wagner & Herbel-Eisenmann (2013).

5 Implications

The various conceptualizations of authority represented in the mathematics teachers' authority diagrams raise a number of important issues, which relate to opportunities. First, the diverse representations illuminated various perspectives teachers worked from when thinking about the authority in their classrooms. Markedly absent were representations of some of the kinds of authority that we might have expected in mathematics classrooms. For example, other than a few of the processes listed in column 2 of Table 1, there were no representations of the kind of logic that is central to proof. As mentioned earlier, the closest item to proof was the mentioning of justification. Yet, further probing would have been required to know what the teacher saw as justifying. As we stated in the opening of the article, we would expect truth and proof to be central to discussions about authority in mathematics, but the results here relate to the language practices of mathematics teachers who foregrounded personal authority, supporting previous findings reported in Herbel-Eisenmann & Wagner (2010).

The variation among the accounts of authority in the diagrams, along with the absence of other possibilities we might imagine, reminds us that any account of the way authority works in a particular situation is contestable. There can be diverse accounts of the authority that focus on different aspects and different ways of looking at relationships in the interactions. There can be no authoritative account of the way authority is working in a particular context though there may be value in discussions about what forms of authority a group of mathematics educators might want to see in mathematics classrooms. The contestability of accounts of authority has implications for the way authority diagrams might be used. In particular, such diagrams give as much (or more) insight into the teacher's perspective on authority as the classroom dynamic being described by the teacher in words.

Second, scholarship has not yet exhausted the useful ways of conceptualizing authority. Some of the images being used by the teachers to represent authority were unlike representations we have read in the literature. Thus there is potential to investigate these and other representations and images, and what they mean to teachers. However, some dominant metaphors and ideas about authority were repeated in the teachers' diagrams and their discussion around the diagrams. For example, teachers talked about authority using possession metaphors, which have been contested by Oyler (1996), yet some of the teachers were struggling to find new ways of thinking about authority while still using common metaphors. We think it is impossible to reconceptualize an idea like authority without reference to old and perhaps unsatisfying images. Reconceptualization needs to be incremental. Thus, we found hope in the teachers who struggled with their drawings and with talking about their drawings.

As an example of the potential in the diagrams to spawn new possibilities for thinking about authority, we will share one example from our reflections on the diagrams as a set. We were particularly intrigued by the way teachers depicted and talked about texts as authorities. We see now that there is potential in differentiating among users' intentions with texts. For example, students often look in the back of a mathematics textbook to check their answers. Students also look at examples in mathematics textbooks, which structure their thinking before their independent work. In the former situation, the authority is consulted after the action. In the latter situation, the authority is consulted prior to the action. Texts that are consulted prior to action seem to have similarities to a canon in literature; they are archetypes. Our use of positioning theory draws our attention to the human interactions in any situation, and thus we ask how classroom interactions position texts as a priori authorities or evaluative authorities to measure alignment or normalcy.

Third, the work of mathematics teachers and their perspectives on authority depend significantly on their personal and professional contexts. Thus, we think it is inappropriate to generalize about what features of authority are the most important to consider in a mathematics classroom. Instead, we suggest that further investigation of authority bear in mind the connections among teacher's personal contexts, their professional contexts, their perspectives on authority, and their pedagogical decisions relating to authority.

Given the inappropriateness of drawing from authority diagrams generalizations about which aspects or representations are most important, we ask what the value of these and other authority diagrams could be. We see potential for research and for education practice. The striking diversity of the teachers' representations suggests to us the likeliness of even more variance among other groups of teachers. Thus further investigation of teachers' authority diagrams is warranted in various contexts. As noted in the introduction, authority structures are often mentioned in analyses of mathematics learning contexts but teachers and students are rarely asked about their perspectives. Our attention to mathematics teachers' perspectives on the authority structures in their classrooms reflects some of the literature and extends beyond it, suggesting the need for much more attention in the research. For example, Amit & Fried (2005) identify a web of authority relations but only identify a few of the sources of authority identified by the teachers in our study. This study is the first to attend to *teacher's* conceptualization of authority made by researchers in their analyses.

As suggested by the experience of the three teachers in the third context, for whom the diagrams were anchors in their reflections and development, we would promote the use of such diagrams for teachers as well as others. Teachers might draw different diagrams to represent the authority relationships in their various classroom contexts or different diagrams to represent the authority relationships in various types of classroom activities or structures within one class. Teachers could compare their diagrams with those of their colleagues and discuss the differences as a way of opening up discussion of differences in their perspectives and their contexts of

practice. Teachers could also invite their students to draw authority diagrams, and thus gain insight into their students' sense of place in mathematics. At least one of the teachers in our study did this. Similarly, in professional development contexts, leaders could use such diagrams as a tool for promoting reflection and opening up conversation among teachers. The subsequent discussions will also give the leader insight into the participant teachers' contexts and ways of thinking. We encourage leaders in professional development to use this and other tools for listening to the teachers with whom they work.

Discussing the value of authority diagrams as a tool for personal reflection brings us back to considering opportunities in research in addition to prompting further research. We have drawn authority diagrams to represent the relationships in our research contexts. This experience helped us understand *our* actions in relation to research participants and also in relation to our field of study. We have, for example, worked on diagrams to represent the authority relationships in the research reported here. We have chosen, however, not to reveal our own diagrams in this article because we think doing so would tacitly privilege particular forms of representation in the teachers' diagrams, forms that we chose to use in ours. The value of the diagrams is not to show one form of representation as being better than another but rather to open up understanding of how educators think about authority. We encourage researchers, as well as teachers, to use authority diagrams as a tool for reflection. We hope to learn more about the way authority works in classrooms and in research as others take up this tool.

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