Learning mathematics in a French Immersion classroom presents a situation with unique language dynamics, the kind of situation in which there is the potential for gaps in linguistic and associated conceptual development. Modality is important in mathematics education because it encodes conjecture and makes possible a space between not knowing and confidence in truth statements. By mapping the form of modality, we investigate grade 10 French Immersion students engaged in problem solving to see how conjecture is encoded. From this we identify issues for researchers and teachers to consider in such settings.

INTRODUCTION

Conjecture is an important part of mathematical exploration. When people explore a mathematical problem together, as with mathematical investigations in classrooms, it is necessary to have a way of suggesting an idea before knowing it is true. Rowland (2000) highlighted the centrality of such conjecture to mathematics, and coined this “space between what we believe and what we are willing to assert” (p. 142) as the Zone of Conjectural Neutrality (ZCN).

As claimed in the Whorfian hypothesis, the language resources available in any given context makes possible certain ways of communicating and thinking about problems, and may render other ways of thinking improbable. For example, Shaffer (2006) explained how deaf children with hearing parents did not develop what she called “The Theory of Mind” because of the absence of modality in their vocabulary. Modality refers to linguistic tools for expressing degrees of certainty. Linguists Martin and Rose (2003) described the effects of modality this way: “it opens up a space for negotiation, in which different points of view can circulate around an issue” (p. 50) – a description that bears close resemblance to Rowland’s ZCN. Once the children in Shaffer’s study developed vocabulary for modality (in American Sign Language) it became clear that these tools facilitated their quick development of this Theory of Mind, which relates to the ability to separate one’s self from his or her ideas.

The phenomenon described by Shaffer connects to our interest in French Immersion (F.I.) mathematics classrooms. Students’ development of mathematical skills and understanding in early years of F.I. contexts would be supported with students’ relatively weak language resources in the language of instruction. Students develop
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more complex language structures later on but their beliefs and dispositions related to mathematics may already be shaped by their early experiences.

For this paper, we analyse the modality used by grade 10 students working in groups engaged in mathematical problem solving. We identify modal structures, first to show that they exist, and second to consider what the particular linguistic choices made by students might suggest about their view of mathematics. Finally we reflect with questions about implications for mathematics educators, both teachers and researchers.

MODALITY

Modality refers to the range of certainty that can be expressed in text, oral or written. One way of expressing degrees of certainty is to use modal verbs. For example, “it must be six” is stronger than “it could be six.” *Must* and *could* are modal verbs. Expressions with more certainty have high modality and those with low certainty have low modality. Bald assertions, which Rowland (2000) called *root modality*, are stronger yet than expressions involving the highest modal verbs. Saying “it is six” does not suggest alternative possibilities. By contrast, “it must be six” suggests a recognition that others may disagree. Any language that draws attention to the possibility of various points of view is deemed in appraisal linguistics as heteroglossic. Bald assertions are monoglossic because no alternatives are indexed, nor is point of view. Statements range from high to low polarity, with modal verbs indicating stances between the extreme positions:

<table>
<thead>
<tr>
<th>Statement</th>
<th>Polarity</th>
</tr>
</thead>
<tbody>
<tr>
<td>it is six</td>
<td>high polarity (root modality)</td>
</tr>
<tr>
<td>it must be six</td>
<td>high modality</td>
</tr>
<tr>
<td>it might be six</td>
<td>modulated polarity</td>
</tr>
<tr>
<td>it could be six</td>
<td>low modality</td>
</tr>
<tr>
<td>it cannot be six</td>
<td>high modality</td>
</tr>
<tr>
<td>it is not six</td>
<td>low polarity (root modality)</td>
</tr>
</tbody>
</table>

Martin and Rose (2005) distinguished among five types of modality, which describe degrees of certainty relating to usuality, probability, obligation, inclination and ability (p. 50). For example, “we sometimes add six” refers to usuality, “it is probably six” refers to probability, “we must use six” refers to obligation, “I might use six” refers to inclination, and “I can use six” refers to ability. With these examples, we show that there are other ways of introducing modality, in addition to using modal verbs. For example, one can introduce the adverb “probably” in various ways to express limited yet strong confidence.

Rowland (2000) classified modality based on the referents. *Alethic* (or logical) modality distinguishes between the necessary and contingent. For example, “the next card could be an ace” can be a statement of fact, despite the modal verb *could*, which often expresses doubt. Here it expresses knowledge that there is a certain chance of the next
card being an ace. This situation corresponds to Martin and Rose’s category referencing probability. Epistemic modality refers to belief. Finally deontic modality refers to obligation. We note that Rowland’s distinction between alethic and epistemic modality is especially important in mathematics. The calculated likelihood of an event is very different from assuredness in belief though these situations employ the same language tools in many cases. Further, we notice that even expressions of deontic modality may be taken as epistemic – for example, in “we must use six” the modal verb must seems to be an expression of belief though must usually refers to obligation.

Mathematics discourse that features modality can be associated with a fallibilist philosophy of mathematics. Such a philosophy underpins certain ways of teaching mathematics, in particular, ways in which students are introduced to problems that they work on without having been given solution methods in advance. Alternatively, mathematics can be expressed as complete and known, as it is in many mathematics textbooks and in classrooms that have been described as traditional. We (Herbel-Eisenmann and Wagner) and others (including Rowland) have written about the relation between modality and approaches to mathematics.

**CONTEXT**

The classroom transcripts used in this paper come from a research project that aims to identify practices that can help mathematics teachers increase their awareness of the role of authority in their teaching. Participant teachers met together for three years to discuss the way authority worked in their classes, sometimes commenting on video or audio excerpts from each other’s classes.

The data used for this paper comes from audio recordings and transcripts of group work from a participant whose mathematics teaching comprised mostly F.I. classes. This set of transcripts comes from her grade 10 class in her second year in the study. She had been noting that the positioning of students was different in small group interaction compared to full class discussion. In particular, she said that she often overheard students expressing their authority more in group interaction, both when she had structured the groups for particular problems and when they interacted while doing independent work. Thus, on her request, we recorded small group interaction on investigative problems. In her remarks on a draft of this paper, she again expressed pleasure in seeing these particular students demonstrating confidence and knowledge.

Working with functions and relations, students were responding to the prompt shown below. This is the English version from the parallel textbook (Knill et al, 1998a, p. 239):

> The E-shapes are made from asterisks.
> (a) How many asterisks are in the 4th diagram? the 5th diagram?
(b) Plot the number of asterisks versus the diagram number for the first 5 figures. Is the graph discrete or continuous?
(c) Does the graph represent a direct variation or a partial variation?
(d) Write an equation in the form $A = ____$ to represent the function, where $A$ is the number of asterisks in the $n$th diagram.
(e) Using the equation to find the number of asterisks in the 55th diagram.

Using the equation from (d), calculate the number of asterisks that contain the 55th figure.

And this is the text from the actual prompt that students worked from (Knill et al, 1998b, p. 239). The diagrams were the same, but called “Figure 1”, “Figure 2” and “Figure 3”:

Ces figures en E se composent d’astérisques.
(a) Combien d’astérisques y a-t-il dans la 4e figure? dans le 5e figure?
(b) Représente graphiquement le nombre d’astérisques en fonction de numéro de la figure pour les 5 premières figures. Le diagramme est-il discret ou continu?
(c) S’agit-il d’une variation directe ou d’une variation partielle ?
(d) Formule une équation de la forme $A = ____$ qui définit la fonction, ou $A$ est le nombre d’astérisques dans la $n$e figure.
(e) À partir de l’équation en d), calcule le nombre d’astérisques que contient la 55e figure.

FINDINGS

First, we describe the modality expressed by Vicki and Alyson (all names here are pseudonyms), who chose to work together. Both Vicki and Alyson expressed most of their ideas with root modality but each employed some modulation. In addition to these expressions, they also asked questions of each other. Authentic questions, like modality, encode heteroglossic discourse because they engage another’s point of view. Here is an excerpt from their discussion, which demonstrates the coexistence of alethic and epistemic stance. The column on the right is a translation of the transcription on the left. The students were using the constant $b$ to refer to the vertical axis intercept of a graph.

Vicki
42 Comment est-ce que tu as sait ça ?
Alors, $b$ est cinq. Je ne comprends pas, mais…c’est cinq.

Alyson
43 Non, c’est trois.

How do you know that? So $b$, it’s five. I don’t understand, but…it’s five.

No, it’s three.
Vicki’s question in turn 42 is heteroglossic, and invited Alyson to respond with alethic or root modality — with logic, or with a root statement that refers to a logical argument. For example, “Figure zero would have three” would have employed alethic modality, and “Figure zero has three” would be a root statement that implies the same logical argument though it would be monoglossic. Instead, she responded with root modality and no engagement of reason. Nevertheless, Vicki seemed to agree that \( b \) is three. Vicki’s question in 42, which asked for logic, was not answered as she seemed to want.

Vicki’s next question in turn 44 invited monoglossic root modality. Instead of replying in the expected way, Alyson inflected her response upward to turn it into a question. A downward or flat inflection with the response “five” would have seemed to address Vicki’s request. Finally, Vicki’s question in turn 46 again invited modality because she asked what Alyson thought \( x \) was. The alternative, “What is \( x \)?” would have invited monoglossic root modality. Alyson avoided answering, perhaps because she did not know the answer or perhaps because she did not understand the question. Nevertheless, her imperative, “You have to teach me” employed high modality to indicate obligation. She made this demand in English. We wonder here whether a demand given in one’s home language is stronger than one given in an additional language. Lunney Borden (2010) observed that Mi’kmaw students were more compliant with their teachers’ demands made in their first language, Mi’kmaq, than with demands made in English.

Following this excerpt, Vicki reviewed some terminology with Alyson using root modality, and thus suggesting that there were no alternatives to this terminology. Perhaps she saw no alternatives because she was indexing an earlier class discussion and thought that all students in the class would use the terminology in the same way. After this review, modality appeared again, but in English (code-switching is a common phenomenon in multilingual classes):

<table>
<thead>
<tr>
<th>Alyson</th>
<th>Est-ce que c’est ça ?</th>
<th>[In French…] Is that it?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vicki</td>
<td>Yeah, I’m thinking it’s…we</td>
<td>[In English…] Yeah, I’m thinking</td>
</tr>
</tbody>
</table>
already got five. I think it’ll work with any. Won’t it?

53 Alyson Like...Is it how many times five goes into it? Like, twice, like three times?

In English... Like...Is it how many times five goes into it? Like, twice, like three times?

54 Vicki Oh no, I don’t think so. Because the last one, the x was 92 remember? Because that was the distance travelled. I think it’s one, because y equals five times one, which is one, plus three. So y equals eight. Right? No, no, ça c’est l’équation! Parce que tu substitues la figure pour le x.

In English... Oh no, I don’t think so. Because the last one, the x was 92 remember? Because that was the distance travelled. I think it’s one, because y equals five times one, which is one, plus three. So y equals eight. Right? Now in French... That’s the equation. Because you substitute the figure for the x.

Here Vicki modulated her earlier result by saying in turn 52, “I think it’ll work.” She used the modal verb think to express uncertainty and, further, she checked by asking, “Won’t it?” Alyson responded with a question and then a root modal, which was modulated by her tone. Vicki’s response (turn 54) used the modal verb think twice. And she checked by asking, “Right?” Though the tone suggests to us that she was not sure of herself, this same question could be used rhetorically. When she returned to French, she moved back to root modality.

We might wonder whether Vicki switched to English to express modality, perhaps showing a lack of linguistic resources to express modality in French. However, in a later monologue she employed a similar structure along with other forms of modality, speaking in French:

64 Vicki Oui! Um, je pense pour astérisque nous peut compter par cinq, et oui, je sais, ça c’est pourquoi j’ai fait [inaudible]. Ça c’est pas une raison.

[inaudible]

[pause]

J’ai lancé ta crayon. ..

Ça peut être, comme la figure, nous peux faire deux carrés pour la chose pour faire [inaudible]. Et pour ça, nous besoin deux carrés pour chaque cinq, alors 12 carrés. [pause]

Oh, oui. Attends. Parce que figure

Yes! Um, I think for the asterisk we can count by fives, and yes, I know, that’s why I did [inaudible]. That’s not a reason. [inaudible]. That’s not a reason.

[pause]

I threw your pencil.

It can be like, the figure, we can do two squares for the thing to make [inaudible] And for that, we need two squares for every five, so 12 squares. [pause]

Oh yeah, wait. Because the figure
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Vicki used a number of modal verbs. In “I think […] we can count by fives” she modulated with think and with can. Think is probably epistemic modulation referring to ability. Can might be taken as alethic, epistemic or deontic. Nevertheless can seems to refer to ability or obligation. Categorizing modality in real episodes is challenging because words are used for various purposes. Further challenges occur because both the speaker’s intention and the listener’s interpretation are relatively inaccessible.

In the middle of the monologue, Vicki moved into higher modality with “we need two squares” and continued this level of modality later with “it has to be a square.” These instances of high modality were not as strongly positive than root assertions would have been, but they demonstrate her awareness of the possibility of a range of certainty within mathematical dialogue.

REFLECTION

Our analysis of this episode at the most basic level represents an existence proof: we see that F.I. students can have a sense of modulation within their mathematics dialogue. From this one instance, we see that Vicki has this sense. Alyson seemed to avoid modulation. So we know from Vicki’s heteroglossic speech that she has the linguistic resources to represent degrees of certainty, and also that she chose to use these tools to represent the way she was thinking about her mathematics. The fact that most of Vicki’s speech was monoglossic does not refute either of our claims about Vicki. Certain ideas were beyond question for her and others were more open to alternate possibilities.

We cannot say from this analysis how prevalent modality tools are for F.I. students, nor can we make generalities about when F.I. students develop these linguistic resources or about how the timing of this development relates to the development of the students’ sense of the nature of mathematics work. The existence and various forms of modality in this one instance open up significant questions that would warrant similar investigation of discourse excerpts from various levels of F.I. mathematics.

If F.I. students develop linguistic resources for modality later than students learning mathematics in their first language, which we think is likely, would their views of mathematics differ? In particular, if students learn mathematics in a new language, and thus with limited vocabulary for modulation, are they more likely to think of mathematics in monoglossic terms? Would they see mathematics as engaging mostly in what Rotman (1988) called ‘scribbling’ tasks (work that can be done exclusive from human interaction) rather than engaging in what he called ‘thinking’ tasks (inclusive work that implies human interaction)?
Considering the development of modality among F.I. students, we also wonder how teacher modelling relates? There are various possibilities for expressing modality in French. For example, how does “Vous devez,” which is usually translated “you must,” compare with “Il faut”? Translations of “Il faut” typically use the modal verb must but there is no subject of the verb. It means something like “it must be that.” Are both forms used in F.I. mathematics classrooms? Do students adopt the same modal expressions as their teachers, or do they perhaps use French versions of typically English expressions? For example, would they say “vous devez” instead of “il faut” because it has a clearer English equivalent? We might wonder which is better, but, more importantly, the distinction warrants consideration of differences in how mathematics is represented — as foregrounding agentic humans or as masking human agency.

Our analysis of this rather brief episode raises for us many questions. Thinking about modality in this rather specialized setting helps us think about it more generally in mathematics learning contexts. For example, what is the role of a mathematics teacher in developing students’ modality? Is it important to be explicit about the role of uncertainty in conjecturing and its centrality in mathematical development? When mathematics teachers hear students using low modality is it taken as an indication of a lack in self-confidence or an understanding of the necessity of hypothesis and hypothesis-testing?

Training additional-language teachers usually involves extensive attention to modality and the typically complex verb forms that encode it. Perhaps F.I. mathematics teachers would benefit from explicit work in this area and F.I. mathematics students as well. Perhaps all mathematics teachers would benefit from this.

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References