MATHEMATICS TEACHERS' REPRESENTATIONS OF AUTHORITY[1]

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Though issues of authority abound in education and schooling, mathematics teachers' perspectives on authority have not been investigated sufficiently. We describe a tool that we developed to initiate dialogue with teachers about authority—using a diagram to represent authority in their classrooms. Using mathematics teacher's authority diagrams, we investigated what sources of authority they represented and how the teachers related these sources to each other. The diversity in their representations showed us that research on authority in classrooms has merely scratched the surface.

INTRODUCTION

Issues of authority abound in education and schooling. For example, at the broad system level, authority appears in the monitoring of students' performance. At the classroom level, authority occurs in the teacher-student relationship. In mathematics education, authority is especially important because of the discipline's characteristic interest in truth and proof. A quantitative investigation of a large body of transcripts from secondary mathematics classes corroborated the prominence of authority in mathematics classroom discourse, but showed that authority structures were commonly contingent on social positioning, rather than reasoning (Herbel-Eisenmann, Wagner and Cortes, 2010). Authority was placed unquestioned in the teacher and in accepted mathematical procedures.

In most research on teaching and teacher education that addresses authority, it is not a central object of inquiry. The few theorizations of authority do not provide insight into how mathematics teachers think about it. Amit & Fried (2005) provided the most substantive work on authority in mathematics education contexts, with theorization that is substantiated with some interviews with students but not with teachers. Knowing how mathematics teachers think about authority is imperative to understanding issues of authority and agency in mathematics classrooms, but we have not found research that focuses on teachers' conceptualization of authority.

In this paper we describe a tool that we have developed to initiate dialogue with mathematics teachers about authority in their classrooms—using a diagram to represent how they think authority works in their classrooms. Our analysis of the diagrams teachers created and discussed in our work together helps answer our question: *How do mathematics teachers think about authority in their classrooms?*

AUTHORITY IN CLASSROOMS

Authority is one of many resources teachers employ for control and has been defined in an educational context as "a social relationship in which some people are granted

the legitimacy to lead and others agree to follow" (Pace & Hemmings, 2007, p. 6). This relationship is highly negotiable. Students rely on a web of authority relations with friends and family members as well as with the teacher (Amit & Fried, 2005). Although Pace & Hemmings and Amit & Fried consider authority to being negotiated between people, tools and artefacts can also be considered authorities, especially in the context of schooling. For example, textbooks play a prominent role in what and how content is taught, especially in mathematics and science classrooms. Additionally, other tools (like graphing calculators) have been shown to be resources students rely on as they consider the correctness of their work. Nevertheless, the role of these resources in the classroom is mediated by teachers (e.g., Herbel-Eisenmann's (2009) analysis of the use of mathematics textbooks in classrooms).

We see the idea of positioning as being important to work on authority because it recognizes that relationships necessarily involve issues of control, authority, and power. These issues appear at many levels, including interactions within a classroom (in one-on-one interaction, small groups and whole-class interaction) and between people in the class and stakeholders outside of it. Harré and van Langenhove (1999) described positioning as the ways in which people use action and speech to arrange social structures. In their theorization of positioning, they showed how clues in word choice or associated actions evoke images of known storylines and positions within that story. For example, a teacher may see herself as a coach while the student sees her as a drill sergeant. Such multiplicity of possible storylines demonstrates that various authority relationships may be envisioned in any particular situation.

Distinction between the actual positioning or authority structures and what people say about these power relations is significant. Using interviews with mathematics students, Amit & Fried (2005) claimed to look at the way authority actually is in classrooms, but one might argue that they were only looking at what students said about authority. Authority is only a conceptualization, however, so like with positioning theory, there is no empirical authority relationship. There are only people's perceptions or attributions of authority. Thus any account of authority is contestable.

METHOD

The primary data in this article are diagrams generated by teachers to describe the way they see authority at work in their classrooms. These data were supplemented by transcripts of our recorded dialogue with teachers, as they described their diagrams and asked questions of each other. The contexts for these dialogues varied, though they are set in research studies oriented primarily around professional development for participant teachers. They were all set in different cities in Eastern Canada. Because of the professional development focus in these contexts, we focused on prompting reflection and discussion among participants, which proved useful for our research question too. In particular we add focus to our research question by asking: 1) What sources of authority do teachers represent in diagrams depicting classroom authority? 2) How do they conceptualize the relationship among these sources?

The first context in which we had teachers draw diagrams was at the outset of our study engaging secondary mathematics teachers in conversation about authority structures in their classrooms. We interviewed each teacher and asked him/her to describe his/her view of authority in his/her classroom. Preceding the instructions for drawing the diagrams, we asked the following questions: 1) What or whom do your students see as authorities in your classroom? 2) How do your students know something is right in mathematics? 3) How do your students know what to do in mathematics? 4) How do you, as a teacher, know what is right and what to do in mathematics? After listening to the teachers' answers to these questions, we drew for each teacher a thick dot on a blank paper or blackboard and said, "This dot is you." We then invited them to use symbols, lines, words, or whatever they needed to show how authority works in their classroom to complete the diagram. As the teachers were drawing, they talked about what they were drawing, and we asked them questions like "What does this arrow mean?" or "Why did you use a dotted line to connect those parts instead of something else?" (We reported on the connections between their diagrams and their unique contexts in Herbel-Eisenmann & Wagner (2009)). The diversity in their diagrams and descriptions prompted us to use this line of questioning in our other interactions with teachers and pre-service teachers, both in and outside of research contexts.

The second research context was a two-day professional development for grades 6-9 mathematics teachers. The session focused on discourse in mathematics classrooms and was led by six mathematics education researchers. The context was different from our first context because the teachers were not alone when making their diagrams. They did not talk about their answers to the four questions about authority, but instead were given time to make notes and reflect on those questions. When drawing their diagrams, they could probably see other's diagrams in their periphery but they did not talk during that time. When the diagrams were complete, they in turn described their diagrams to the group. For each description, the group was invited to ask questions and make comments. Questions of clarification were encouraged – "Why did you … in your diagram?" The third research context was another two-day session in a different city, modelled on the two-day session described above.

Our analysis began with our identifying and categorizing depicted 'sources' of authority in the diagrams. Whether or not teachers talked about these 'sources' as showing authority, we call them 'sources' because they are potential sources. Even a passive person might be called a source of authority because one's acquiescence to another person in a relationship is part of what gives that other person authority. Items we do not take as sources of authority include the arrows and lines, which we took as representing connections between the sources of authority in the diagrams (though we recognize that this exclusion could be problematic).

Our interpretation of what the symbols were supposed to represent and how teachers thought about these objects is informed by the teachers' accompanying discussion about their diagrams. Connecting this commentary with the diagrams also highlighted

the extensive choices teachers made when drawing, all of which relate to the teachers' conceptions of the sources of authority. Even our choice to represent a person as a dot (and teachers' choice to follow suit, using dots to represent people) is a significant choice that relates to a complex array of alternative possibilities in relation to the teacher's experiences, though the dot itself appears very simple.

We have been asked why we asked teachers to represent themselves with a dot. We could have left it open as to how or where they represent themselves in their diagrams. In our view, using a non-dimensional dot to represent the self helps focus the diagrams on the relationships and interaction more than on personal identity. If a teacher was to consider how she would represent herself, we would expect the focus of her attention in the exercise to be significantly different. With a non-dimensional dot, her representations of herself would more likely be in terms of her relationships with others, rather than on identity markers (e.g. gender, clothing).

After looking at the objects in the diagrams we focused our attention on the symbols used to indicate relationship among the objects. For example, many teachers used arrows to connect objects. The placement and direction of such arrows suggests a teacher's sense of which objects connect and of how they connect.

It appeared to us that most teachers drew their sources of authority first and then connected them, but we believe that some of the teachers began their drawing with symbols suggesting relationships and added in sources afterwards. Nevertheless, our interaction with the teachers facilitated our understanding of what in their diagrams were of primary importance to them and what their symbols meant to them.

TEACHERS' VIEWS ON AUTHORITY: SOURCES OF AUTHORITY

We begin our findings with a focus on differences in who and what teachers included in their diagrams, which we refer to as sources of authority. Table 1 lists the items shown in the 34 diagrams with their frequencies (how many different diagrams they appear in). We see more diversity of sources of authority in these diagrams than what we have seen reported in the literature. For example, we have seen self, students, families, and peers as sources of authority in the literature but have not seen professional teaching organizations cited as authority sources in classrooms. Additionally, the category we label as "processes/actions" encompass a range of sources that we have rarely seen in work on authority. Because there is such an extensive list in each column, we highlight a few ideas about each of the columns rather than discuss each item teachers included.

Following our instructions to the teachers, every diagram included the self, the dot in the centre. Our choice to specify that the teacher be represented as a dot likely influenced the teachers' choices of how to represent others in their diagrams – for example, many teachers used dots to represent students and other people as well. Some teachers used symbols other than dots to represent individuals. Perhaps this choice suggests that teachers wanted to say something about different identities of students. (They could use dots for each person if they wanted to focus on the

interactions.) Some of these teachers talked about wanting to indicate that students made choices in their classrooms. In these cases, the different depictions of students might say less about individual student identities and more about the teacher's understanding of the choices a student could make on any particular day.

people	processes/actions	classroom objects	disciplinary artefacts
self (34)	questioning (4)	textbooks (10)	math curriculum (10)
students (22)	communicating (3)	black/whiteboard (7)	prior skills (4)
family (7)	discussing (3)	calculator (4)	ideas (3)
other teachers (4)	directing (2)	computer (3)	prior problems (3)
administration (3)	giving feedback (2)	desks (3)	expected answers (2)
department of education (2)	answering (1)	manipulatives (3)	classroom rules (1)
professional learning community (2)	confirming (1)	books (2)	daily routine (1)
church (1)	comparing (1)	materials (2)	methods (1)
groups (1)	discovering (1)	posters (2)	prior experiences (2)
NCTM (1)	disrupting (1)	resources (2)	questions (1)
school board (1)	estimating (1)	ruler (2)	roles (1)
sport teams (1)	focusing (1)	handouts (1)	tests/exams (1)
tutor (1)	guiding (1)	paper (1)	"There is no math god" (1)
vague others (1)	instructing (1)	technology (1)	topic (1)
	investigating (1)		
	justifying (1)		
	memorizing (1)		
	planning (1)		
	positioning (1)		
	practicing (1)		
	prompting (1)		
	questioning (1)		
	raising hand (1)		
	understanding (1)		

Table 1: items represented in 34 authority diagrams and their frequencies

Even among the diagrams that use dots for people, there are distinctions between kinds of dots, as we showed in our early reporting on authority diagrams (Herbel-Eisenmann & Wagner, 2009). Dawn and Jill[2], whose diagrams are given in that reporting, used open and closed dots to distinguish between kinds of people. Dawn also used an x to represent the student as distinct from teachers who were dots. Mark, whose diagram is also discussed in our previous reporting, used sizes of dots to represent the relative weight of authority ascribed to individuals. Not all students were depicted as equal, demonstrating an awareness of complex differences in relationships—there are not only teacher-student relationships but there are many kinds of student-student authority relationships.

Other teachers distinguished between students in other ways. For example, Dallas (Figure 1) used rectangles to represent his students (perhaps these are desks with metonymic connections to the students), and showed some as having questions and others as not engaging. Another teacher used a dot to represent a group of students

instead of a single student. We wonder what this view of students as collectives instead of as individuals means for his teaching.

Thirteen teachers focused on actions or processes (column 2 in Table 1) by using symbols and words (which are also symbols) to explicate what happens in the interactions among the sources of authority. These thirteen teachers came up with 24 different processes to describe the nature of the interaction. This diversity indicates that teachers' views on authority differ significantly and that one could expect other teachers making diagrams to include more processes yet. Sometimes teachers sitting close together when drawing seemed to borrow ideas from each other, but the diversity of responses is evidence that the teachers had significantly different points of view and the desire to express these views.

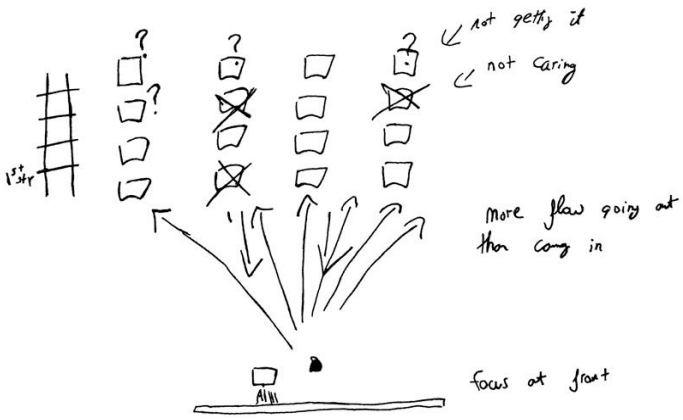

Figure 1: Dallas' authority diagram

Manipulatives, which are concrete objects used to help model mathematical ideas (column 3) appeared in a number of diagrams as sources of authority and garnered some discussion when teachers presented their diagrams to each other. When asked to say more about manipulatives as an authority in her diagram, Rochelle said,

> We're doing algebra now so they need the tiles on the desk with them. If they're not sure if they got it right they can go to their tiles and they can use them to play around, and make sure it [the answer] works. And if your tiles don't give you the same answer, well, that is giving you feedback—to see, to see if they have the right answer.

Manipulatives are inanimate objects, but Rochelle attributed authority to them, much like looking up answers in the back of a textbook or checking one's work on a calculator. Students can "go to the manipulatives" to test their ideas. However, the manipulatives are controlled by the student and are used as a tool. It is interesting to us that Rochelle and two other teachers (including Louise, see Figure 2 in the next section) represent manipulatives as separate from the self though they are extensions of the self. This split is reminiscent of the split self described by Rotman (2008). He described how people doing mathematics embody different roles within themselves; the 'thinker' instructs the 'scribbler' what actions to do to inform further thinking. A student's work with manipulatives or with a calculator is an example of the scribbling (or rote action) performed to explore one's thoughts, which seem to operate separately in a conversation within oneself. Similarly, the 'ideas', depicted by three teachers, may reflect the 'thinker', the other side of the self, described by Rotman.

Of the disciplinary artefacts (column 4) teachers referenced, not surprisingly, the mathematics curriculum was the most prevalent source mentioned. The suggestion

that "prior" skills, problems, and expectations might be sources of authority was interesting because these items suggest attention to what has previously been described in the literature as "common knowledge" (Edwards and Mercer, 1987) in classrooms. Other disciplinary artefacts appeared as part of the set of the social norms that guided the classroom work such as rules, routines, and roles.

TEACHERS' VIEWS ON AUTHORITY: POSITIONING OF SOURCES

In addition to the various depictions of sources of authority in the diagrams, we found variation in how the sources were arranged. For example, seven of the 34 teachers arranged their diagrams to depict the physical arrangements of their classrooms. For instance, Dallas' diagram (Figure 1) shows the arrangement of desks with himself and the blackboard in front. Louise (Figure 2), one of the three teachers who depicted manipulatives as sources of authority, showed how the desks were arranged in groups in her classroom and showed one wall with some postings on it and a computer centre up against it. She positioned herself in the centre of the class. The arrows, however, are relatively figurative. She talked about herself at the centre moving outwards: "the teacher is in the centre and circulates to as many students as possible." She also said that students made choices about how and where to work on their problems thus there are different kinds of arrangements of her students, but these differences depict structures of their interaction (or social positioning) more than physical positioning.

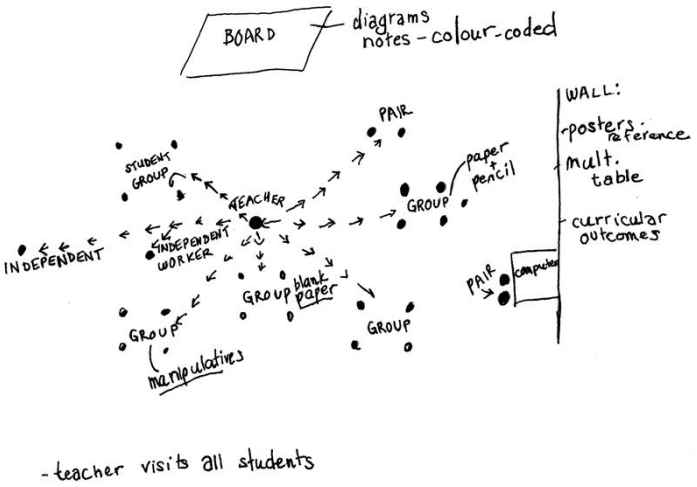

Figure 2: Louise's authority diagram

By contrast, Mark did not use arrows in his diagram. He did, however, use lines, which he said indicated his movement throughout the class. He talked about balance without mentioning direction (no beginning and no end): "I'm really all over the place." Mark and some other teachers had their dot at the front of the room in their diagrams, and talked about their position at the front of the room. Though they did not say that this was significant in terms of how authority works in mathematics, we suggest that this physical positioning, who stands in front of the others, has a powerful impact on social positioning.

Most significantly, 27 of the 34 teachers did *not* organize their diagrams to reflect the arrangement of their classrooms. They saw their diagrams more metaphorically. In most diagrams, the teachers used lines, arrows and other symbols to help represent the metaphoric relationships among the objects in their diagrams.

We believe that the metaphors used to connect the sources of authority are as important as the sources themselves. Of the 34 authority diagrams, 24 had arrows

connecting people to other sources of authority but there were differences among the arrows. For example, Dawn said her arrows represented someone looking to an authority, and Jill used arrows to indicate the flow of authority in communications, thus Dawn's arrows and Jill's appear to be in opposite directions. Also, Louise (Figure 2) used broken arrows, which she did not explain in discussion. These differences suggest different ways of thinking about authority.

Other metaphors appeared as well. Dallas (Figure 1) depicted a ladder and described a relationship that had him making decisions about what students should do. With this description, Dallas garnered sympathy from at least one other teacher who helped him finish his sentences:

Wagner: What's the railway track?

Dallas: Oh that's actually a ladder. I have some kids that don't get math. They just shut down when I ask them a question, but if I can say, "Okay, but you know this because we talked about it the other day. You know this, you know that." So then we go up another step and they do good. But it's me trying to break it up evenly. So that's the strategy.

Woman: That's how they know what to do. You lead them through it.

Dallas: Yeah, with each individual one it's—

Woman: It's exhausting.

Dallas: Yeah, that's the word.

Jean's (Figure 3) diagram employed another metaphor. The mirror and window may not look very central to the diagram, but they were the first aspects of the diagram about which she talked. Before describing the images that represented what she called 'influences' on students, she said, "I thought it was important that kids have a window to see forward and also a mirror to see a reflection of themselves, and that it was important to reflect." She did not say what artefacts students might use as prompts for reflection. Perhaps they used their manipulatives, as described by other teachers, or perhaps they relied on memories. Nevertheless, her comments remind us of Skovsmose's (2005) suggestion that both background and foreground are important for working with students. Jean's metaphor also reminds us of Gutiérrez (2011), who also used a mirror/window metaphor to say that students should be able to use mathematics to look out at the world but also to see and recognize themselves.

Yet another metaphor was described by Joanne (Figure 4). She saw authority as something that can be passed from one person to another but in the short time given to think about this she was still seeking imagery that recognized the fluidity of relationships and authority.

> If I'm in the centre, I drew a spiral, meaning that I'm everywhere in the classroom. I looked at it as communication. And authority is kind of passed on to the students through giving them, I guess, the understanding or the ability to problem solve and to work through and to help each other. So it leaves me anywhere along here you can find

students. And there's arrows going every which way so it's been passed between the students and between myself. And I talked about the ripple effect. Even though the authority might be given to me to teach these students then ultimately I'm passing it on to them to conduct their own learning. […] I drew those [radial lines] kind of like a radar […]. It's kind of pulsing out, [like a] spider web, interconnected. I was going to have the web and the radar and the ripple effect all happening at once. The important thing is I'm not anywhere per se and the students aren't in any exact position. It's all transient.

Oyler (1996) challenged possession metaphors for authority. Such metaphors suggest authority is like a finite resource and that one person's increased authority implies someone else's loss. Joanne's description demonstrates for us her struggle to conceptualize authority differently from the dominant possession metaphor.

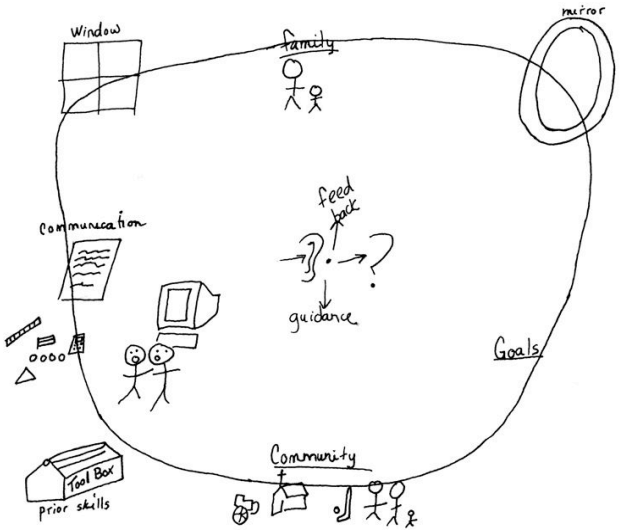

Figure 3: Jean's authority diagram

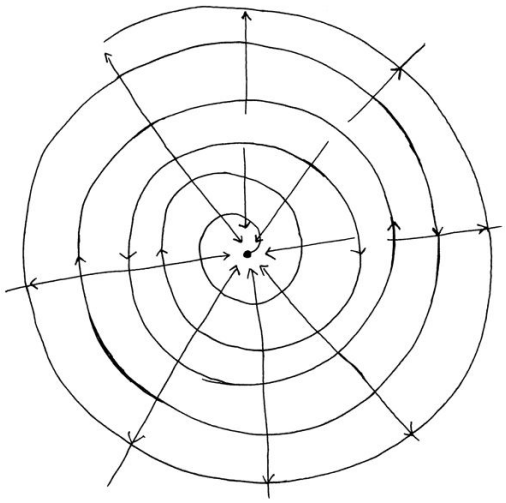

Figure 4: Joanne's authority diagram

IMPLICATIONS

The diverse representations of authority generated in the diagrams illuminate various perspectives teachers work from when thinking about the authority in their classrooms. This reminds us that any account of the way authority works in a particular situation is contestable. There can be diverse accounts and there can be no authoritative account of the authority working in a particular context.

It is also clear from the diversity in the diagrams that scholarship has not yet exhausted the useful ways of conceptualizing authority. Some of the images being used by the teachers to represent authority were unlike representations we have read in the literature. Thus there is potential to investigate these and other representations and images, and what they mean to teachers. We note that the teachers in our research work in the same geographic region, and thus we wonder whether there would be even greater diversity represented by teachers coming from more diverse cultures.

NOTES

1. This research was supported by the National Science Foundation (Grant No. 0347906) and the Social Sciences and Humanities Research Council ("Positioning and Authority in Mathematics Classrooms"). Opinions, findings, conclusions and recommendations do not necessarily reflect the views of the granting bodies.

2. All names of teachers are pseudonyms.

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