LANGUAGE REPERTOIRES FOR MATHEMATICAL AND OTHER DISCOURSES

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We start with the stance that it is important for educators to understand students' language repertoires in relation to characteristically mathematical conceptualizations and processes. The data in our study of grade 3 to 11 students' language repertoires for conjecture led our attention to competing discourses in the classroom and the consideration that the mathematical language repertoires are also repertoires for friendship, competition, etc. In this paper we use a framework for identifying authority structures in mathematics classrooms to focus on formative communication acts, and then we consider how each act might serve a purpose for positioning the students involved in terms of each discourse that we identify as in play.

Keywords: Classroom Discourse, Equity and Diversity, Problem Solving, Instructional Activities and Practices

Introduction

Because mathematics is mediated through and by language, it is important for educators to understand students' language repertoires for mathematical conceptualizations and processes. As part of a large-scale research project focused on identifying specificities of students' language repertoires, especially in contexts of mathematical investigation, we sought to identify connections between their ways of talking about conjecture and what they think their expressions mean.

In this paper, we use one episode from our data to problematize our fundamental research question. Positioning theory helps us understand the way students' communication acts connect to a variety of discourses, including mathematics. There are many discourses enacted in any classroom context. In addition to mathematics, we found mischief, romance, play, hunger, and more at work. In short, we cannot understand students' communication about mathematical processes without understanding that these acts are also part of their repertoires for the other discourses in play. We argue that the connections among the discourses indexed by students' communications may give us insight into what mathematics is for these students.

Positioning and Discourses

Position metaphors are often used to illustrate the way people relate to each other. Van Langenhove and Harré (1999) have described positioning as the ways in which people use action and speech to arrange social structures. According to this theorization, in any interaction, the participants envision known storylines to help them interpret what is happening. These storylines may be conscious or not. They are contested explicitly or implicitly. A powerful aspect of this theory is its radical focus on the immanent, its rejection of the transcendent. In other words, it considers real only that which is present in the interaction and rejects the power of exterior forces.

In an analysis of the way this theory was taken up in mathematics education research, Wagner & Herbel-Eisenmann (2009) noted that exterior forces, such as the discipline of mathematics, may be myths, but they can be taken as real in classroom or other interactions because teachers and others may be viewed as representatives of these exterior forces. The classical triad developed by the progenitors of the theory (e.g., van Langenhove & Harré, 1999, p. 18), which connected a speech act to a storyline and a positioning, was reconfigured by Herbel-Eisenmann, Wagner, Johnson, Suh, & Figueras (2015, p. 194), as shown in Figure 1.

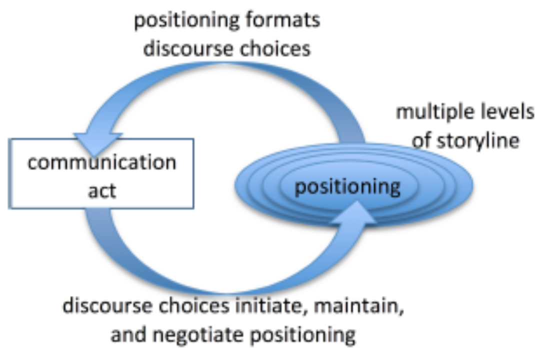

Figure 1. Illustrating positioning. (Herbel-Eisenmann et al., 2015, p. 194).

They layered storylines to emphasize positioning theory's claim that multiple storylines may co-exist in an interaction. And they used arrows to highlight the dynamic interaction between a communication act and the exterior storyline—a communication act initiates, maintains, and negotiates positioning within a storyline, and this positioning formats communication acts. For us, this recursive relationship is reminiscent of Foucault's (1982) description of discourses—"practices that systematically form the objects of which they speak" (p. 52). Thus we will use the term *discourse* instead of *storyline.*

For our analysis in this paper we elaborate the diagram into three dimensions (Figure 2).

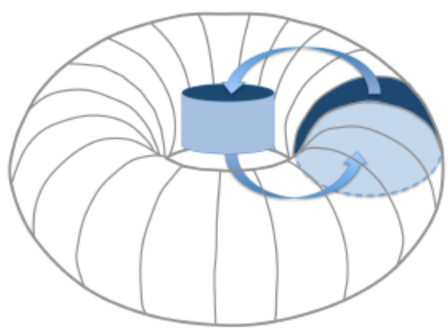

Figure 2. Extending the positioning model into three dimensions.

We are motivated by our data to avoid foregrounding any one discourse. Thus we position a range of discourses in relation to a communication act. Each discourse may be seen as a slice of the torus though the lines of demarcation between discourses would not be so clear as they are in the diagram; the various discourses are interconnected. A communication act appears as a cylinder passing through the torus. The cross section cut of this 3D figure would appear about the same as Figure 1. The communication act (a slice of the cylinder, which would appear as a rectangle in cross-section) interacts with a discourse (a slice of the torus, which would appear as an ellipse). The arrows show how the communication act constructs the discourse while the discourse constructs the communication act.

Connecting communication acts to discourses

Because positioning theory focuses on people's rights and obligations in interactions, we choose a conceptual frame developed in mathematics education that makes distinctions among such structures. Working from a large-scale quantitative analysis of the communication in mathematics classrooms, Wagner & Herbel-Eisenmann (2014) distinguished among four authority relationships. To identify <u>personal authority</u>, one looks for "evidence that someone is following the wishes of another for no explicitly given reason" (p. 875). Linguistic clues include the presence of *I* and *you* in the same sentence, exclusive imperatives, closed questions, and choral response. To identify discourse as authority, one looks for "evidence that certain actions must be done where no person/people are identified as demanding this" (p. 875). The strongest linguistic clue is the presence of modal verbs that suggest necessity—e.g., *have to, need to, must*. To identify discursive inevitability, one looks for "evidence that people speak as though they know what will happen without giving reasons why they know" (p. 875). The modal verb *going to* is a strong indicator of this structure. Finally, to identify personal latitude, one looks for "evidence that people are aware they or others are making choices" (p. 875). Linguistic indicators include open questions, inclusive imperatives, and indicators of someone changing their mind—for example, *I was going to, could have*.

We emphasize that we identify these authority relationships on the basis of particular communication acts, and try to avoid reading intention. For example, a student may say or do something because s/he thinks that is what the teacher wants, but we look for communication acts that explicitly indicate this authority structure. The intention may be significant, but our attention to language repertoires compels us to look for the communication acts that underlie or motivate this intention. We take the stance that the sense that a student is doing what a teacher wants comes from one or a series of communication acts that set up this discourse.

In this paper we work through a transcript identifying authority relationships. We focus on instances where these relationships appear to change and on aspects that are resilient to change. At the same time we identify exterior discourses that are referenced (We acknowledge that there are further discourses at play that are never explicitly addressed in the transcript, or that are addressed in a way that we do not recognize). In addition to the mathematics, we identified the following discourses in this particular group's work: school work, hobby/game playing, our research agenda, competitiveness, romance, affect in identity, clothing, friendship, body image, and physical/material resources. We considered referring to these other discourses as "distractions" but realized that this word is subjective; yes, romance may distract from mathematics, but mathematics may also distract from romance. Thus we think of the whole set of discourses, including mathematics, as competing and intertwined discourses. In our analysis, for each significant communication act, we ask how the specificities of that act might serve the person's interests within the various discourses. How is mathematics positioned? How does this act position the friendships? How does it project body images? etc.

We chose this particular group of three grade 10 students (approximately 15-years-old) because they made progress mathematically though their teacher did not have high hopes for them working well together. They were given a page with the following task and some images of cubes and cut up cubes (e.g., Figure 3): "A cube was painted red, and then cut into smaller cubes, 3 x 3 x 3. How many of the small cubes have no red faces? How many have 1 red face? 2 red faces? 3 red faces? 4 red faces? 5? 6? How about a cube cut into 4 x 4 x 4? Or 5 x 5 x 5? Or 10 x 10 x 10? Or n x n x n?" They were also given a set of twenty-seven solid white cubes snapped together in a 3 x 3 x 3 configuration. It was possible for students to pull the set apart and reorganize the cubes but they still had to visualize what sides would be painted and what would not.

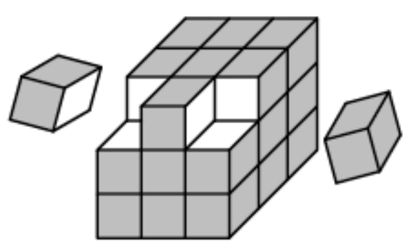

Figure 3. Cube task images.

Applying the Framework

The task involved working in a group and working mathematically with an object that could only be visualized. This kind of investigation work invites (perhaps 'requires') students to decide how they will organize their investigation and themselves while doing the investigation. Steven set the tone:

3 Steven: All right boys, well, let's get to work. "A cube was painted red and cut into smaller cubes, 3 by 3 by 3…"
4 Peter: I don't have one.
5 Steven: You don't have a sheet? [turning to a researcher…] Um, excuse me? He don't have a sheet. [turning back to the group] Okay.

This exchange establishes the positioning in the group. Steven's "let's get to work" (turn 3) suggests personal authority, similar to the way a teacher might demand work from students; he told the others what to do and did not tell them why it would be a good idea or why he had the authority to do so. First we consider how this positions mathematics. It appears that mathematics comprises performance at the behest of someone in authority, performance of certain kinds of procedures or processes (involving shape and number in this case, and probably more generally). This authority structure does not align with conventional views of mathematics being a bastion of reason. Steven also indexes the friendship discourse with "All right boys" (turn 3), and positions himself as a leader in this group of friends (however, we were told by the teacher that these boys were not friends). Connecting these discourses, mathematics is positioned as something one can use to establish authority in friendship, and vice-versa—friendship can be used to establish authority in mathematics. The other discourses we identified in this episode were not yet explicit. We pick up the interaction when the group begins to engage with the task:

12 Steven: twenty-seven have no red on them. Oh wait, no, that's wrong. [*he holds the large cube and points at small cubes when counting]* 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, … oh, fuck.
13 Doug: No look…
14 Steven: It's like, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, …
15 Doug: All this would, here, here, and here [*pointing*].
16 Steven: Just the outer layer. Just the outsides.
17 Doug: Yeah all the other.

Steven was in control of the cubes while Doug and Peter watched him. Though it could appear that they were not paying attention, we know they were paying attention because of what they said later. In turns 12 to 17, there are no expressions suggesting personal authority, discourse as authority, or personal latitude. By deduction, the dialogue might suggest discursive inevitability, but we ask whether it fits the description of this category. There are no instances of *going to*, which is the marker identified by Wagner & Herbel-Eisenmann (2014). However, the students are working as though there is a correct answer that they will identify once they have worked enough at the task (which aligns with the description of this category). Linguistically, this is achieved in a number of ways. Steven counts outside of sentence structure (turns 12 and 14). This suggests that there is only one way to count the objects; no one would count them differently. If they had thought there are different ways of counting, they would qualify their counting with, for example, "if we count [this way] we would get […]." Furthermore, Doug's expression, "this would" (turn 15) suggests that he knows what will happen.

We now ask how these communication acts position the students with the discourses we have identified in the entire session. First, because of the orientation we bring to the research (as mathematics educators) we are attentive to the expression “this would” (turn 15) because it is a way of indicating a generalization. The expression will appear multiple times later in the transcript as part of a clearer generalization. In the case above, the sentence is incomplete so it is difficult to see Doug’s generalization. Second, we found no linguistic references to other discourses. There were markers of some of them throughout the interaction, however; Doug’s shirt indexed his hobby, the clothing of all three of them suggested that they identify as male, their proximity and body language could relate to friendship discourses, their physical context connected them to school discourses, our presence in the room and the recording devices potentially reminded them of the research discourse, etc. Nevertheless, the grammar of their interaction, specifically the ways their language suggested inevitability within mathematics, eclipsed the other discourses that potentially could draw their intention. Indeed, their attention was drawn to the recording devices, a girl in another group, and other “distractions” at times in this interaction. This idea of inevitability suggests that there is one way of counting, for example, but it also draws attention away from other discourses—it seemed to be inevitable that they would count those blocks in the way presented in the task. As the conversation continued, there is a change away from discursive inevitability:

18 *Steven*: I wonder if you take it apart? Like this? [*As he takes the cubes apart they do not separate as he intended.*] Fuck. All these, the outer, if you look on the inside… these two sides don’t, these two sides don’t, take that apart, these two don’t…
19 *Doug*: Yeah, but also…
20 *Peter*: Hey Steven, can I see the cube for a sec?
21 *Steven*: One sec.
22 *Peter*: Okay.
23 *Doug*: Once you take it apart you’re saying all these would have red, but what he’s asking is that if you take an individual cube, so this was right here right, and like
24 *Steven*: Oh.
25 *Doug*: This part would have red, this part would have red, this part would have red on it, so those three sides… er… yeah, three sides would have red.
26 *Steven*: So how many of them wouldn’t?
27 *Doug*: Only three cubes… one cube
28 *Steven*: If this whole thing was painted red, how many of these would have no—that’s what he’s asking, how many of these? Okay, so it would go like this, one, two, oh yeah, so it would be these sides right? It would be this side, this side, this side, this side, and this side. So yeah, it would be like, how many of them? It would be this… no, this side wouldn’t have…
29 *Doug*: No, that side wouldn’t
30 *Steven*: All right, this side wouldn’t and this side wouldn’t.

Starting in turn 18, Steven’s “I wonder” demonstrated an awareness that he can think about the cube structure in different ways. This suggests personal latitude because he expressed awareness of his choice about ways of seeing. This change in authority structure seems to be contagious. Doug’s response “Yeah, but also” (turn 19) references personal latitude too. He agreed with Steven and identified that something could be added. And Peter then asked for the cube (line 20), indicating awareness that there is choice about who holds the cube. Steven’s response in line 21 then indicates awareness that he can choose whether to honour Peter’s request, and then Peter complied, likewise acknowledging that he can agree or disagree with Steven’s request for a little more time. This led to the beginning of a conjecture (though not clearly worded) from Steven: “If you take…” (turn 23). Turns 24 to 30 are replete with the word *would*, which we indicated earlier as a marker of discursive inevitability. However, in this case the word has different meaning, specifically a mathematical justification, because it is paired with the subordinate conjunction *if*.

This flurry of personal latitude may be an example of interlocutors picking up discourse patterns from each other. Applied linguistics literature has identified this phenomenon in terms of people picking up each other's words, which is a little different than picking up an authority structure. Positioning theory discusses the possibility of picking up structure from others, as the theory describes (using the language of first-order, second-order, and third-order positioning) how people follow or resist positioning established by others. Nevertheless, as we ask what happened to switch the dialogue from a stream of discursive inevitability to this flurry of personal latitude, we note the possibility that the authority structure was part of the students' repertoires the whole time just waiting to be triggered. It appeared to be triggered by Steven's insight, but one could say this too was triggered by the task inviting a certain kind of thinking. It could also have been triggered by certain classroom norms that honour this kind of interaction. There are lots of possibilities.

Steven's "If you take" (turn 23) is the beginning of a generalization in which he imagined stripping the cube of the outside layer of small cubes, which would leave one bare cube in the middle. The conjunction *if* is strongly associated with conjectures and generalizations throughout the interaction of this group especially in its pairing with *would*: "If …, then … would…" though the word *then* tended to be omitted. This and the other ways of expressing the students' choices to see the cube in different ways supports the development of their mathematical insights, and thus constructs mathematics as a discipline in which people develop insight by making choices about how to see things. These communication acts that index personal latitude can make similar impacts on the other discourses though it is not straightforward to map them because this part of the interaction was focused on mathematics. Nevertheless, the former hierarchy within the friendship discourse, in which Steven had the role of telling his "boys" what to do, was opened up to allow for the others to exercise agency. The finite physical resources were a factor in this negotiation of the object of attention though it is clear from their dialogue to come that both Doug and Peter were manipulating the cube in their imagination as they watched Steven manipulate the physical cube.

Discussion

Our research interest was initially on student language repertoires for mathematics. Our data has led us to connect this to some of our previous work that recognized and identified a range of discourses at work in mathematics classroom contexts (c.f. Andersson, 2011). In reflection we return to the repertoires. We remember that whatever repertoires people have for mathematical thinking and action, these repertoires serve purposes in other discourses as well. In fact, participation in other discourses that share linguistic resources with mathematics informs one's mathematical thinking. And vice-versa; participation in mathematics informs one's thinking and action in other discourses that share linguistic resources.

Thus we argue that mathematics teachers (with the support of mathematics education researchers) would be well-served to develop further understanding of how the characteristically mathematical expressions appear in students' discourses outside the classroom. Such insight would inform the ideas about mathematics the students would have when hearing and using the expressions in mathematics. For instance, two of the students in the focus interaction in this paper were "gamers." We are not very familiar with gamer culture, but we know that participants are in constant interaction with each other, and they are confronted with a continuous and fast-paced stream of choices that impact their progress in the game. Thus the students in our focus interaction would be using the "If … then …would …" construction in their gaming just as they used it in their mathematics. And we wonder how these two environments connect to each other for these students.

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